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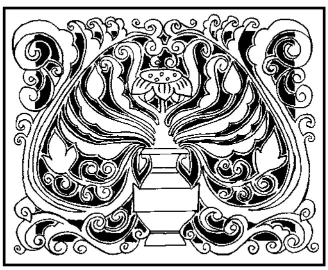

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COVER: 'The Indian Scientific Heritage'
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Traditional Wisdom

उत्तिष्ठत जाग्रत प्राप्य वरान्निबोधत । ***Arise! Awake! And stop not till the goal is reached!***

The Scientific Spirit

September 2007
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न ता मिनंति मायिनो न धीरा व्रता देवानां प्रथमा ध्रुवाणि ।
न रोदसी अद्रुहा वेद्याभिर्न पर्वता निनमे तस्थिवांसः ॥

Not men of magical skills, not men of wisdom impair the primeval laws of the gods. Ne'er may the heaven and earth, which know no malice, nor the fixed hills, be bowed by sage devices. (Rig Veda, 3.56.1)

यथा शिखा मयूराणां नागाणां मणयो यथा ।
तद्वेदाङ्गशास्त्राणां गणितं मूर्धनि स्थितम् ॥

As are the crests of peacocks and gems on the hoods of serpents, so is mathematics at the very forefront of the Vedic auxiliaries. (*Vedanga Jyotisha*, 4)

प्रायेण मनुजा लोके लोकतत्वविचक्षणाः ।
समुद्धरन्ति ह्यात्मानमात्मनैवाशुभाशयात् ॥
आत्मनो गुरुरात्मैव पुरुषस्य विशेषतः ।
यत्प्रत्यक्षानुमानाभ्यां श्रेयोऽसावनुविन्दते ॥

Generally speaking, persons endowed with the capacity to investigate the truth of things lift themselves from the evils of instinctive life by their own discriminative power. One's guru is oneself, especially for human beings, for they are able to raise their lives to a cultural level through observation and inference. (Bhagavata, 11.7.19–20)

उत्पादकं यत्प्रवदन्ति बुद्धेरधिष्ठितं सत्पुरुषेण सांख्याः ।
व्यक्तस्य कृत्स्नस्य तदेकबीजमव्यक्तमीशं गणितं च वन्दे ॥

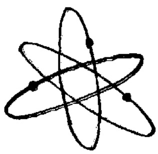
That which the wise speak of as the generator of knowledge when one is grounded in it, which is the unmanifest source of all that is manifest—to that unmanifest Ruler and to mathematics, I bow down.
(Bhaskaracharya, *Bijaganita*, 'Mangalacharana', 1)

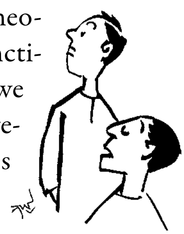
यथाम्बुजवनस्यार्कः प्रदीपो वेश्मनो यथा ।
प्रबोध्यस्य प्रकाशार्थं तथा तन्त्रस्य युक्तयः ।

As is the sun to a forest of lotuses and a lamp to a house, so is logic (indispensable) in illuminating scientific texts.
(*Sushruta Samhita*, 'Uttaratantra', 7)

THIS MONTH

The Indian Intellectual Tradition has long remained inadequately explored and poorly understood. It has often been thought of as non-rigorous, mystical, and impractical. This number should help you reassess this myth even as it takes you on an intellectual journey through the rough terrains of physics and mathematics.





To many of us, scientific theories seem to have the sanctity of religion. Rarely do we stop to ponder over the reality of scientific theories or the validity of observation and reason as tools for the advancement of science. Only a closer look at the bases of science will reveal the true sources of its power. Dr N Mukunda, Centre for High Energy Physics, Indian Institute of Science, Bangalore, shows us how in **Philosophy of the Physical Sciences**.

Doing mathematics may not quite be everyone's cup of tea, and **The Philosophy of Mathematics** may appear too intimidating a subject for non-specialists. Swami Sarvottamanandaji, Dean of Research, Ramakrishna Mission Vivekananda University, Belur, shows us why it is not.

Prof. Vijaya Kumar Murty, Chair, Department of Mathematical and Computational Sciences, University of Toronto, recapitulates the important contributions of Indian mathematicians in **A Brief History of Indian Mathematics**.

Rasa is a key concept in Indian aesthetics. The many connotations, associations, and practical implications of this term have been lucidly elucidated in **The Philosophy and Aesthetics of Rasa** by Dr Makarand Paranjape, Chairperson, Centre for English Studies, Jawaharlal Nehru University, New Delhi.

Dr Kapil Kapoor, former Professor of English and Concurrent Professor, Sanskrit Studies, Jawaharlal Nehru University, New Delhi, concludes his exposition of the **Philosophy of Language in the Vaiyakarana Tradition** with a discussion of the fluidity of meaning, its relation to verbal intuition, and the ontological roots of language.

Swami Durganandaji, Ramakrishna Mission Vidyalaya, Coimbatore, examines the Indian view of the cosmos and its evolution in the concluding section of **The Cosmos in Western and Indian Thought**.

The fourth instalment of **The Many-splendoured Ramakrishna-Vivekananda Vedanta** focuses on some interesting references to Sri Ramakrishna, Swami Vivekananda, and Sri Sarada Devi in contemporary texts on Mahatma Gandhi. The author, Dr M Sivaramkrishna, is former Head, Department of English, Osmania University, Hyderabad.

Dr Amartya Kumar Dutta, Associate Professor of Mathematics, Indian Institute of Science, Kolkata, provides an interesting look into an ancient text, ***Bijaganita* of Bhaskaracharya**.

Grisha Perelman: The Madness of Knowledge is a fascinating portrait of a contemporary mathematician by Br. Brahmachaitanyaji, Ramakrishna Mission Vivekananda University, Belur.

The Indian Intellectual Tradition

In his essay on 'The Nyaya and the Architectonic of Logic', P T Raju observes: 'Gautama [the founder of the ancient Indian school of logic called Nyaya] felt that salvation (*niḥśreyas*) can be obtained only if the right effort is made. Our effort can be right only if it is in accordance with reality; for life has to be planned according to reality. But then we have to know what reality is. Our knowledge has, therefore, to be right and logically valid. There is no other way to determine reality than experience and logically valid forms of knowing. When the forms and their methods and, through them, reality are properly understood, then only can salvation be possible. So Gautama enunciated sixteen categories of logic, epistemology, and argumentation as means to salvation.'

Nyaya as a system of philosophy has always been both pragmatic and realistic. Its pragmatism is reflected in its refusal to develop a purely formal framework of deductive logic totally divested from empirical considerations, as has been done by modern mathematical logic. Its realism is grounded in the pluralistic world view assiduously developed by its cognate system, the Vaisheshika. The Vaisheshikas were probably the earliest analytical philosophers in India. They attempted a complete description of objective reality in terms of six conceptual categories (later increased to seven).

Moreover, none were as 'unrestrained in their speculations', and none 'such powerful critics of time-worn prejudices as the followers of *Kaṇāda* [the Vaisheshikas]'. Gautama mentions doubt, aim, empirical examples, general truths, premises, hypothetical reasoning, and conclusions as essential components of his framework of logic. Doubt and aim provide the incentive, empirical observations and general truths the material, and premises and hypothetical (or counterfactual conditional) reasoning the instruments for fresh knowledge. It was this spirit of inquiry and freedom of thought that was responsible for much of the vigour and vitality associated with ancient Indian thought and culture.

If the Nyaya-Vaisheshika schools refused to let go of their empirical moorings, the ancient Indian mathematicians made rapid strides into the realm of abstract reasoning by developing the decimal place-value notation with zero that not only allowed them great facility in handling large numbers and complex computations—including those involving irrational and negative numbers and surds—but also facilitated the development of symbolic algebra and later of analytic trigonometry and calculus.

Intellectual vigour and creativity are not of much practical value if they do not get translated into technological innovation. The theories of modern science have a fascinating grip on the contemporary mind because of the amazing power they grant us to manipulate nature and secure better longevity, comfort, and physical connectivity, and the broadening of our intellectual horizons that has followed as a consequence. A thriving Indian manufacturing industry right up to the eighteenth century, a sophisticated tradition of medicine and surgery based on astute observations, and the excellence of ancient and medieval Indian handicraft, architectural design, shipbuilding, and iron works are proofs of the practical face of Indian thought.

With such sound intellectual traditions, why did we miss the Industrial Revolution in the late eighteenth century? The prime reasons were (i) loss of touch with our intellectual heritage, (ii) isolation from the global community, and (iii) political instability and colonial subjugation. All of these factors are virtually non-operational today. We can, therefore, well expect a vigorous flowering of Indian thought in the coming decades.

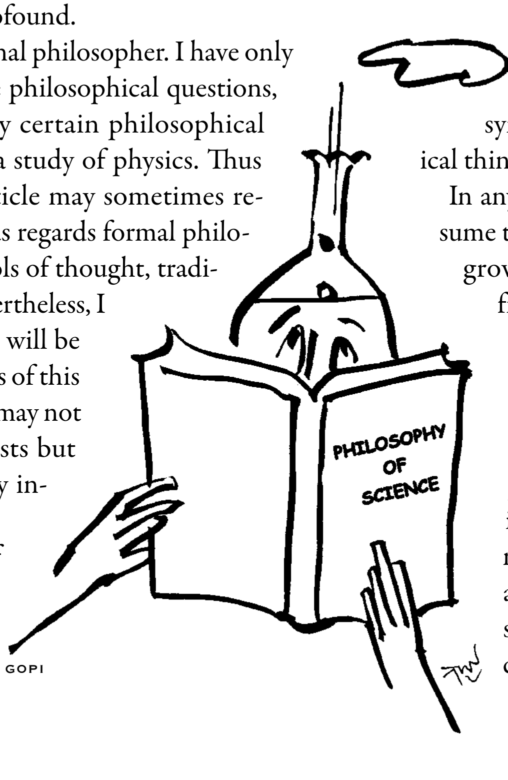
Philosophy of the Physical Sciences

Dr N Mukunda

The various philosophical traditions of the world form an important part of the intellectual and cultural achievements of the civilizations which produced them. Typically, their roots go back thousands of years—as in the cases of India and Greece. There is in them much poetic imagery and logical and deep thinking, as well as a sizeable speculative component. In contrast, modern science as we know it developed barely four hundred years ago, in the seventeenth century, arising in the main out of the combined efforts of Copernicus, Kepler, Galileo, and Newton. It was only then that the importance of controlled experiments and careful and systematic quantitative study of natural phenomena was clearly recognized. However, in spite of these great differences in age, at least in the Western tradition the interactions between modern physical science and philosophy have been deep and profound.

I am not a professional philosopher. I have only been attracted to some philosophical questions, and been impressed by certain philosophical systems, as a result of a study of physics. Thus the content of this article may sometimes reveal a sense of naivety as regards formal philosophical matters, schools of thought, traditions, and the like. Nevertheless, I hope that what follows will be of interest to the readers of this journal, most of whom may not be professional scientists but would still have a lively interest in these matters.

It may not be out of place to mention here some contrasting attitudes to the possible roles and value of philosophical thinking that are evident in the developments in physics over the past century. As a consequence of the European continental tradition, the general writings of the two discoverers of quantum mechanics—Werner Heisenberg from Germany and Erwin Schrödinger from Austria—show great familiarity with and interest in various philosophical systems of thought, from the Greeks onwards. While the writings of Niels Bohr and Albert Einstein also often have a philosophical bent, their references to formal systems of philosophy tend to be fewer, but nevertheless important. In contrast, when the focus of work in the new physics shifted from Europe to the US around the middle of the twentieth century, this regard for general philosophical thinking among the leading professional physicists does seem to have weakened. Typical statements of Richard Feynman and Steven Weinberg, for instance, display a certain degree of disdain, or certainly a lack of sympathy, for the value of philosophical thinking in the physical sciences.



In any case, in the present account I assume that there is value in looking at the growth of modern physical science from a 'philosophical point of view', though it may require some degree of maturity as well as sympathy to adopt this attitude.

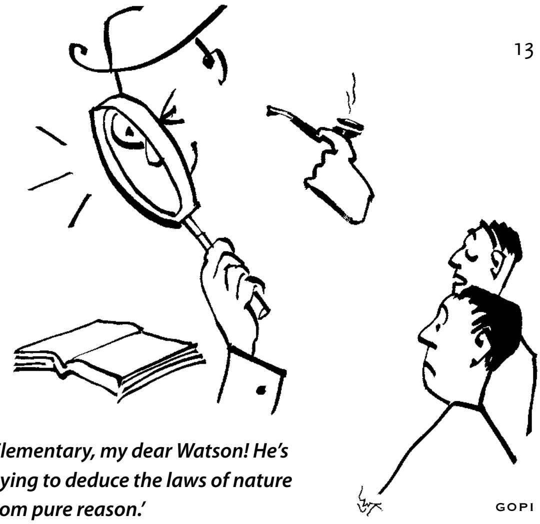
We may say for our present purposes that philosophy of science is generally concerned with the nature of knowledge, the way we acquire it, the meaning of understanding, and the evolution of concepts, all in the context of the phys-

ical sciences. It may in addition be ultimately concerned with an appreciation of our place in nature. Philosophy of science deals with the understanding of natural phenomena and how this understanding is achieved, with the general features common to the various branches of science, and with the interdependence of these branches. It is more interested in the overall pattern of natural laws than in the details of any particular area of science.

Our aim will be to come up to the modern era in physics, and to see what it has taught us with regard to questions of a philosophical nature. Along the way we shall briefly review some historical developments and ways of thinking or schools of thought, both in philosophy and in physical science. We will consider how concepts are created, how they grow, and how they have sometimes to be greatly modified or even abandoned. Naturally, developments in physics will be covered in slightly greater detail than those in formal philosophy.

Rationalism and Empiricism

In our account of the beginnings of science and philosophical thinking we go back to Greek times. The major creative period, lasting about four hundred years, began with Thales of Miletus (c. 624–c. 546 BCE) and included, among many renowned thinkers, Pythagoras (c. 580–c. 500 BCE), Anaximander (610–c. 545 BCE), Democritus (c. 460–c. 370 BCE), Leucippus (fl. 5th cent. BCE), Plato (427–347 BCE), Aristotle (384–322 BCE), and Euclid (fl. c. 300 BCE). In the early period, with Thales, there was a strong impulse towards, as Benjamin Farrington puts it, 'a new commonsense way of looking at the world of things … the whole point of which is that it gathers together into a coherent picture a number of observed facts *without letting Marduk* [the Babylonian Creator] *in*.' The attempt was to deal with nature on its own, not bringing in mystical or mythical leanings. To quote from Heisenberg: 'The strongest impulse had come from the immediate reality of the world in which we live and which we perceive by our senses. This reality was full of life, and there was no good reason to stress the distinction between matter and mind or between body and soul.'



'Elementary, my dear Watson! He's trying to deduce the laws of nature from pure reason.'

Thales was familiar with the knowledge of geometry developed by the Egyptians, the basic facts of static electricity, and the magnetic properties of lodestone. Later, Democritus and Leucippus propounded the atomic concept of matter, not in a casual manner but based on careful reasoning. However, it goes without saying that philosophical thinking in these early times had a considerable speculative content, and there were others such as Plato and Aristotle who later strongly opposed the atomic hypothesis. This should come as no surprise at all, since as late as the end of the nineteenth century there were influential figures—Ernst Mach and Wilhelm Ostwald—who were still opposed to the idea of atoms. This idea finally triumphed only thanks to the heroic efforts of Ludwig Boltzmann, and Einstein's work on Brownian movement.

The knowledge of geometry brought by the Greeks from Egypt was perfected and presented in an axiomatic form by Euclid of Alexandria around 300 BCE. The fact that this subject could be presented as a deductive system—a large number of consequences or theorems following logically from a very few 'self-evident' axioms or 'obvious' truths—must have made a deep impression on the Greek mind. It led in course of time to the idea that the behaviour and laws of nature could be derived from pure reason, without the help of direct inputs from experience. This was the so-called *rationalist* philosophy of science, which lay in stark contrast to the initial

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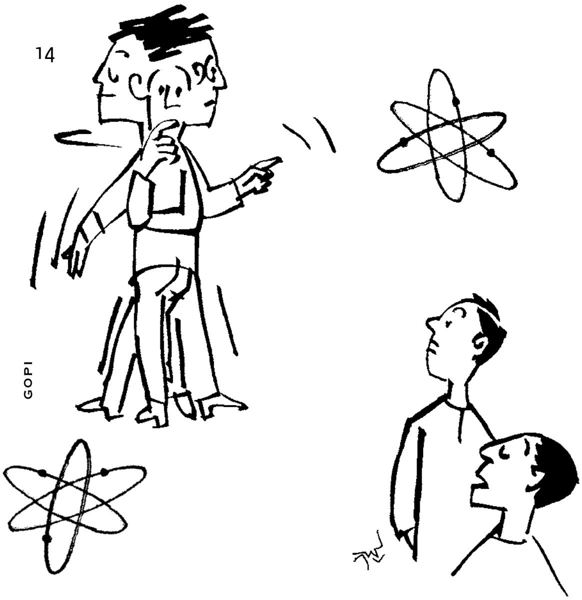


'Even in the early twentieth century there were eminent scientists opposed to the idea of atoms. This idea finally triumphed only thanks to the heroic efforts of Ludwig Boltzmann, and Einstein's work on Brownian movement.'

empiricist approach of Thales and Democritus. Plato held that 'knowledge of Nature does not require observation and is attainable through reason alone'. Before Plato, Pythagoras too espoused this point of view, other illustrious followers being Aristotle and, in much later times, René Descartes, Wilhelm Leibniz, and Benedict de Spinoza. One may say that this rationalist philosophy accords a privileged position to human beings in the scheme of things.

The opposite—empiricist—point of view holds that knowledge comes ultimately from experience of phenomena and not from reason. As we saw, this was the attitude of both Thales and Democritus; and in later centuries it was revived by Francis Bacon and carried forward by John Locke, George Berkeley, and David Hume as a reaction to the rationalist view on the European continent. We shall return to some of these contrasting philosophies later, only noting now that empiricism goes with a more modest attitude towards our place in nature.

From Galileo and Newton to Kantian Philosophy

Modern science emerged in Europe during the Renaissance—the reawakening of classical ideals in arts, literature, and philosophy during the fourteenth to seventeenth centuries, brought about by a combination of social, political, and religious factors. This is not the place to go into this crucial advance in any detail, but we note that it occurred against the background of a liberating intellectual and philosophical atmosphere to which many—including Descartes, Leibniz, and Spinoza—contributed.

Empirical Advances • Nicolaus Copernicus initiated the movement away from a human-centred view of nature with his heliocentric model of the solar system, and Francis Bacon showed the way to freedom from reason alone as the source of all knowledge. Indeed, Bacon said of Aristotle: 'He did not consult experience as he should have done … but having first determined the question according to his will, he then resorts to experience, and … leads her about like a captive in a procession.' Copernicus's work, as well as Kepler's discovery of the three laws of planetary motion during the years 1609–19, was but preparation for what was to come in the work of Galileo and Newton.

Galileo, rightly regarded as the founder of modern science, not only discovered the law of inertia in mechanics, the kinematic description of motion, and the law of free fall, but also stressed the importance of performing controlled experiments, of quantitative measurement, and of the use of mathematics in expressing experimental results. He stated this last point with particular emphasis, saying about the 'book of nature': 'It cannot be read until we have learned the language and become familiar with the characters in which it is written. It is written in mathematical language.'

It was Isaac Newton, born the year Galileo died (at least by one calendar), who completed the work initiated by Galileo and his other illustrious predecessors, and paved the way for the systematic scientific investigation of physical phenomena over the succeeding centuries. We can say that without Newton's crowning achievements, this tradition—the Galilean-Newtonian world view—would not have been securely established. Speaking of the importance of what Galileo and Newton achieved, Max Born says: 'The distinctive quality of these

great thinkers was their ability to free themselves from the metaphysical traditions of their time and to express the results of observations and experiments in a new mathematical language regardless of any philosophical preconceptions.'

Scientific Method • Newton expressed clearly his views on the independent and absolute natures of space and time, stated his three laws of motion for material bodies as axioms, enunciated his law of universal gravitation, and established mechanics as a deductive system. His whole approach and accomplishments made explicit and clear all the steps in the chain of scientific work: observation and experimental data → analysis using mathematics → discovery and enunciation of fundamental laws → further mathematical deduction → predictions to be tested by new experiments. As he put it: 'To derive two or three general Principles of Motion from Phaenomena, and afterwards to tell us how the Properties and Actions of all corporeal Things follow from those manifest Principles, would be a very great step in Philosophy, though the causes of those Principles were not yet discover'd.'

Absolute Space and Time • For the purpose of developing mechanics, Newton invented the calculus. In his presentation he adopted the Greek attitude to geometry and the style of Euclid. Thus he converted knowledge obtained inductively from (refined!) experience—extension from the particular to the general—into a deductive style of presentation. From his laws of motion and universal gravitation, all the empirical laws of Kepler and Galileo followed as logical mathematical consequences. His clear statements about the natures of space and time were of critical importance at this juncture. They mark an important phase in our understanding of these key components of nature, and as we emphasize later, this understanding is never final but develops continually 'in time' as we gather more and more experience. Who better than Einstein to express all this: 'It required a severe struggle [for Newton] to arrive at the concept of independent and absolute space, indispensable for the development of theory. Newton's decision was, in the contemporary state of science, the only possible one, and particularly the only fruitful one. But the subsequent development of the problems, proceeding in a roundabout way which no one could then possibly foresee, has shown that the resistance of Leibniz and Huygens, intuitively well-founded but supported by inadequate arguments, was actually justified. … It has required no less strenuous exertions subsequently to overcome this concept [of absolute space].'

Theory and Experiment • In Newton's work we see a confluence of the inductive and deductive methods, each playing its due role. There was a unification of celestial and terrestrial gravitational phenomena, and many previously intractable problems became amenable to analysis and understanding. At one point he went so far as to claim that he made no hypotheses—'*Hypotheses non fingo*'—hinting at pure empiricism; but this actually shows that modern science was still young. As Einstein aptly said: 'The more primitive the status of science is the more readily can the scientist live under the illusion that he is a pure empiricist.' Today the level of sophistication of the physical sciences is such that every worthwhile experiment is heavily dependent on previous and current theory for its motivations, goals, methods, and analysis.

Over the course of the eighteenth century, the Galilean-Newtonian approach to physical science was amazingly successful. It was applied to problems of celestial mechanics or astronomy, fluid dynamics, and elastic media among others. A distinguished line of mathematical physicists—Leonhard Euler, Joseph Lagrange, Pierre Simon de Laplace, and many others—took part in this endeavour. At one point Lagrange complained that, after Newton, there was nothing left to be discovered! Towards the end of the century, the laws of static electricity and magnetism also fell into the Galilean-Newtonian pattern.

Thought as a Synthetic A Priori • Around this time, the philosopher of the Enlightenment, Immanuel Kant, was so impressed by these successes of the Galilean-Newtonian approach that he created a philosophical system to explain or justify them. We

mentioned earlier the contrasting rationalist and empiricist schools of philosophy. Kant tried to bring them together and offered an explanation of the triumphs of Galilean-Newtonian science along the following lines. He distinguished between *a priori* and *a posteriori* forms of knowledge—respectively in advance of, and as a result of, experience of nature—and between two kinds of statements: *the analytic*, which are empty (such as definitions and statements of a logical nature), and *the synthetic*, which had nontrivial content and could in principle be false. He saw two paths to knowledge about nature—that which is a priori, and that which results from experience. Some of the basic physical ideas underlying Galilean-Newtonian physics, which were actually the results of long human experience and experiment, were regarded by him as synthetic a priori principles. Thus they were claimed to be available to us innately—as a result, one might say, of pure reason—and were necessarily valid and obeyed by natural phenomena. Some of these synthetic a priori principles were the separate and absolute natures of space and time, as expressed by Newton; the validity of Euclidean geometry for space; the law of causality; and later on even the permanence of matter and the law of conservation of mass. In effect, Kant took the knowledge of physical phenomena available in his time and made some of it necessarily and inevitably true and binding on nature. These synthetic a priori principles were present in our minds before any experience of nature; they were thought of as preconditions for, rather than results of, science.

Kant's attempt was made about two centuries ago, and today it is clear that it was tied to his age and to the science of his time. Schrödinger characterizes well the impulse that lay behind Kant's attempt: 'One is very easily deceived into regarding an acquired habit of thought as a peremptory postulate imposed by our mind on any theory of the physical world.' We will shortly look at some of the ways in which physical science has gone beyond Kant's framework, and will describe a fascinating new way of understanding the origin of synthetic a priori principles of thought.

Physical Science in the Nineteenth and Twentieth Centuries

Fields as Distinct from Matter · At the start of the nineteenth century the fields of optics, electricity, and magnetism were separate from one another and from mechanics. Chemistry was a distinct discipline. But over the century many advances were made, which we can only briefly describe here. An early step forward was in the understanding of the nature of light. Thomas Young's experiments on interference brought the wave theory of light back into favour, as against Newton's corpuscular ideas. This was carried forward and firmly established by Augustin Fresnel. Then, as a result of fundamental experimental discoveries by Hans Oersted, André Ampère, and Michael Faraday, the concepts of time-dependent electric and magnetic fields came into being. There were things in nature in addition to and distinct from matter.

Meanwhile, celestial mechanics continued to record stunning successes. Perhaps the most striking example was the prediction by both John Adams and Urbain Le Verrier, based on Newtonian mechanics and gravitation, of the existence of a new planet, Neptune, to account for the observed discrepancies in the motion of Uranus. In 1846 it was found exactly where the astronomers were told to look. (However, a later similar attempt to trace discrepancies in the motion of Mercury to a perturbing planet Vulcan was unsuccessful. The answer came from an entirely unexpected direction—general relativity.)

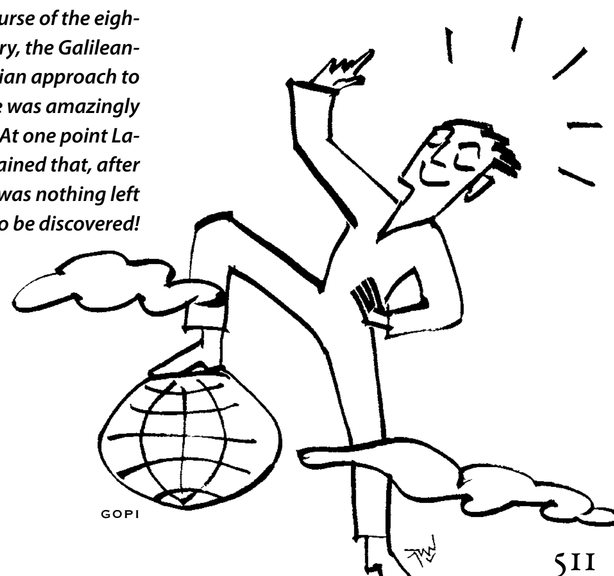
Electromagnetism and Light · After Faraday's powerful intuition had led to the idea of electric and magnetic fields, James Maxwell put all the known laws in the subject of electricity and magnetism into a coherent mathematical form. He then found an important discrepancy, saw the way to correct it, and was thus led to his comprehensive classical unified theory of electromagnetic phenomena. A prediction of this theory was the possibility of self-supporting electromagnetic waves whose speed when calculated turned out to be exactly the known speed of light. Then Maxwell iden-

tified light with these waves, and optics became a part of electromagnetism. During this period, following Fresnel's work, it was believed that the propagation of light needed a material medium, the so-called luminiferous ether, and this concept was taken over by Maxwell as well.

Non-Euclidean Geometry • In the area of mathematics, the subject of geometry witnessed a major advance. We saw that Kant in his philosophy had made Euclidean geometry an inevitable or inescapable property of physical space—it was a synthetic a priori principle. Within mathematics, for centuries the status of one of Euclid's postulates—the fifth one, the parallel postulate (that there is exactly one parallel to a given line through a given point)—had been repeatedly studied: was it logically independent of the other postulates or a consequence of them? During the first half of the nineteenth century, three mathematicians—Karl Gauss, Nikolai Lobachevsky, and János Bolyai—independently showed that it was a logically independent statement. It could be altered, allowing one to create logically consistent alternatives to Euclidean geometry. Thus was born within mathematics the concept of non-Euclidean geometry, which, as we will soon see, was to enter physical science just under a century later.

Statistical Physics • Over the latter half of the nineteenth century, statistical physics and statistical mechanics became established as foundations of thermodynamics. Thus by the century's end the principal components of the physicist's view of the world were Newton's mechanics, Maxwell's electromagnetism, and statistical ideas and thermodynamics.

Relativity • The important departures from the Kantian picture of physical science—from the framework Kant developed to justify the successes of Galilean-Newtonian ideas—came one by one with the revolutionary theories of twentieth-century physics. First came the special theory of relativity, the resolution of a clash between Newton's mechanics and Maxwell's electromagnetism. It turned out that Newton's views of separate and absolute space and time, and the Galilean transformations that go with them, were incompatible with Maxwell's electromagnetic equations. These equations led to a profoundly different view of the properties of space and time. What special relativity achieved was to make clear these properties, show that there was no need for ether as a carrier of electromagnetic waves, and then amend Newton's mechanics of material particles to make it consistent with electromagnetism. The earlier separateness and individual absoluteness of space and time—included among Kant's synthetic a priori principles—gave way to a unified view in which only a combined space-time was common to and shared by all observers of natural phenomena. However, each observer could choose how he or she would split space-time in a physically meaningful way into separate space and time. The earlier absoluteness of the concept of simultaneity was lost, and now varied from observer to observer. For each observer, though, space continued to obey the laws of Euclidean geometry. Special relativity took one step beyond the Kantian framework—now only a combined law of conservation of matter and energy was valid, not separate laws for matter and for energy. (*To be concluded*)



Over the course of the eighteenth century, the Galilean-Newtonian approach to physical science was amazingly successful. At one point Lagrange complained that, after Newton, there was nothing left to be discovered!

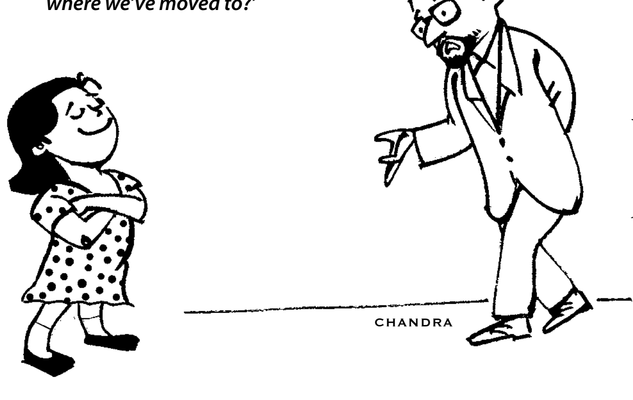
The Philosophy of Mathematics

Swami Sarvottamananda

> 'Ah! then yours wasn't a really good school,' said the Mock Turtle in a tone of great relief. 'Now at OURS they had at the end of the bill, "French, music, AND WASHING—extra."'
>
> 'You couldn't have wanted it much,' said Alice; 'living at the bottom of the sea.'
>
> 'I couldn't afford to learn it,' said the Mock Turtle with a sigh. 'I only took the regular course.'
>
> 'What was that?' inquired Alice.
>
> 'Reeling and Writhing, of course, to begin with,' the Mock Turtle replied; 'and then the different branches of Arithmetic—Ambition, Distraction, Uglification, and Derision.'
>
> —*Alice's Adventures in Wonderland*

The following story is told about the reputed mathematician Norbert Weiner: When they moved from Cambridge to Newton, his wife, knowing that he would be absolutely useless on the move, packed him off to MIT while she directed the move. Since she was certain that he would forget that they had moved and where they had moved to, she wrote down the new address on a piece of paper and gave it to him. Naturally, in the course of the day, he had an insight into a problem that he had been pondering over. He reached into his pocket, found a piece of paper on which he furiously scribbled some notes, thought the matter over, decided there was a fallacy in his idea, and threw the piece of paper away. At the end of the day, he went home (to the old Cambridge address, of course). When he got there he realized that they had moved, that he had no idea where they had moved to, and that the piece of paper with the address was long gone. Fortunately inspiration struck. There was a young girl on the street and he conceived the idea of asking her where he had moved to, saying, 'Excuse me, perhaps you know me. I'm Norbert Weiner and we've just moved. Would you know where we've moved to?' To this the young girl replied, 'Yes Daddy, Mummy thought you would forget!'

'Would you know where we've moved to?'



The world of mathematics is beautifully reflected in *Alice's Adventures in Wonderland*—a world of ideas, where absurdity is a natural occurrence. Mathematics takes us to a world of ideas away from the ordinary, so much so that the archetypal mathematician is typified by the absent-minded professor. In fact, the world of mathematics is an imaginary world, a creation of brilliant minds who live and thrive in it. Mathematics, and the mathematicians who live in its abstract world, alike create a feeling of unworldliness in the common mind. Mathematics is itself abstract; more so is the philosophy of mathematics—the subject of the present article.

Why Study the Philosophy of Mathematics?

Before we enter the subject, we must answer some questions: What is the utility of studying the philosophy of mathematics? And what specifically is the utility in the context of a journal dedicated to Vedanta?

The word *philosophy* is derived from the Greek *philo-sophia*, 'love of wisdom'. Thus, in essence, philosophy as a subject tries to supplement our knowledge by finding out what is knowable and what is

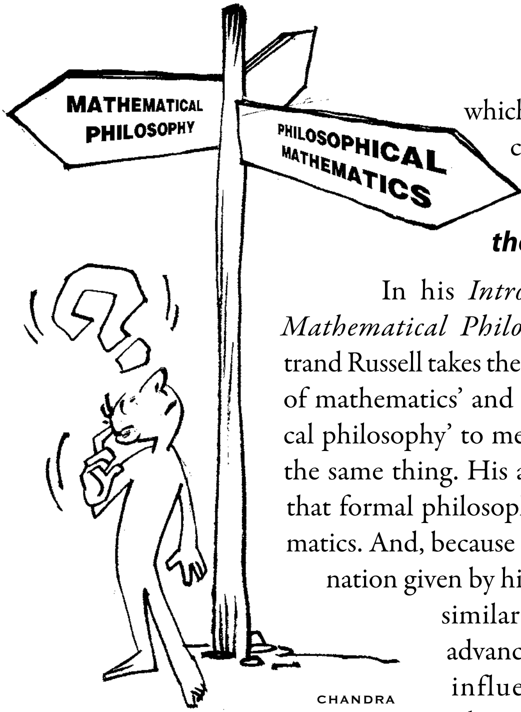
not; just as, in essence, logic as a subject deals with what is provable and what is not, ethics with what is right and what is wrong, aesthetics with what is beautiful and what is ugly, and religion with what is good and what is evil. Vedanta deals with what is real and what is unreal, and asserts *satyam-shivam-sundaram* as a triune entity—that which is real is also good and beautiful. So if we view Vedanta from this angle, then it is religion, ethics, aesthetics, and philosophy—all rolled into one.

The philosophy of mathematics deals with metaphysical questions related to mathematics. It discusses the fundamental assumptions of mathematics, enquires about the nature of mathematical entities and structures, and studies the philosophical implications of these assumptions and structures. Though many practising mathematicians do not think that philosophical issues are of particular relevance to their activities, yet the fact remains that these issues, like any other issue in life, do play an important role in shaping our understanding of reality as also in shaping the world of ideas. This is attested to by the fact that both the ongoing scientific revolution and the concomitant phenomenal rise of technology borrow heavily from the progress in mathematics—a dependence that can be seen throughout the evolution of civilization by the discerning mind.

The importance of mathematics can be judged by the fact that it is used in every walk of life—and this is no overstatement. It is invariably present wherever we find the touch of rational thought. It is the ubiquitous guide that shapes and reshapes our thoughts and helps us in understanding ideas and entities, both abstract and concrete. Moreover, the foundations of mathematics are rock solid. Never has a mathematical position needed retraction. Even in physics, considered a glamorous field in present-day society due to its numerous applications, one finds scientists backing out from positions they held some years earlier. But it is not so in mathematics. Once a mathematical truth is discovered, it seems to remain a truth for eternity. Why is this so?

Contrary to common belief, the real importance of mathematics does not rest in the fantastic theorems discovered; it is in the way mathematics is done—the mathematical process or methodology. It is this that is the matter of our careful scrutiny. Physics has its own methodology too, which is of equal importance. Though it may not appear obvious, both streams stress equally their respective methodologies more than the laws, theories, and hypotheses—that is, the content of physics or mathematics—that they discover or propound. That is one of the chief reasons why there is no crisis in scientific circles when one scientific theory fails and another takes its place.

Contrast this with the philosophies of old, particularly those which were not based on the firm foundation of logic. There the methodologies, the facts and theories, the lives and teachings of the proponents, and, to a lesser extent, the mythologies and cosmologies, were so intermingled, with no clear cut demarcations between them, that systems stood or fell as a whole. It was a favourite technique of opposing schools of thought to point out a single fallacy or discrepancy somewhere in a gigantic work: that was enough to invalidate the whole philosophy. Seen in this light, the strange method of proving the supremacy of one's philosophy that is often seen in Indian philosophical dialectics—through intricate and abstruse arguments as well as ludicrously naïve squabbling—is not likely to surprise us. There will be much to gain if we incorporate the logic of mathematics and the methodology of physics into our classical philosophies, and give up the esoteric dependence on classification, enumeration, categorization, and obfuscation. We need both the fine edifice of logic and the firm foundation of methodology, because most of the Indian darshanas are not mere speculative philosophies but are also empirical—they have many elements of philosophical realism. Of course, the contribution of the Indian philosophies in the realm of mind and abstract thought is enormous. Equally important are the bold proclamations of the rishis about consciousness and transcendental realities,



which are beyond criticism.

Defining the Term

In his *Introduction to Mathematical Philosophy*, Bertrand Russell takes the 'philosophy of mathematics' and 'mathematical philosophy' to mean one and the same thing. His argument is that formal philosophy *is* mathematics. And, because of the explanation given by him as well as similar arguments advanced by other influential people, traditionally, works on mathematical philosophy also deal with the philosophy of mathematics, and vice versa. But a more commonsensical differentiation between these terms may be made thus: Mathematical philosophy is essentially philosophy done mathematically, hence falling within the purview of mathematicians, whereas philosophy of mathematics deals with the philosophical issues in mathematics, something that is to be done by philosophers. Philosophy of mathematics, as we treat the subject in this article, is indeed philosophy taking a look at mathematics, and therefore is not the same as mathematical philosophy.

Thus, we shall only try to look at answers to abstract questions related to mathematics—the form, language, and content of mathematics; the nature of mathematical concepts; and the truth and reality of mathematical discoveries and inventions. Philosophy of mathematics, hence, is truly the metaphysics of mathematics—*meta-mathematics*, the higher knowledge of mathematics. 'Normal mathematics', on the other hand, deals with the relatively mundane, the concrete, the useful, and the visible.

The Subject Matter

Let me clarify a misconception. We are apt to think that when we talk about the philosophy of mathematics we are dealing with all that is abstruse and complicated. Nothing can be further from the truth. It is the simple facts and elementary theorems of mathematics that pose the greatest difficulty to philosophical understanding, by virtue of their fundamental nature, a nature with essential properties which we unknowingly take for granted. To illustrate the point, we list here some of the questions that philosophy of mathematics examines and the classical philosophical domains to which they belong:

- Are numbers real? (Ontology)
- Are theorems true? (Rationalism)
- Do mathematical theorems constitute knowledge? (Epistemology)
- What makes mathematics correspond to experience? (Empiricism)
- Is there any beauty in numbers, equations, or theorems? (Aesthetics)
- Which mathematical results are astounding, elegant, or beautiful? (Aesthetics)
- Is doing mathematics good or bad, right or wrong? (Ethics)
- Can non-human beings do mathematics? (Philosophy of Mind)
- Can machines do mathematics? (Artificial Intelligence)

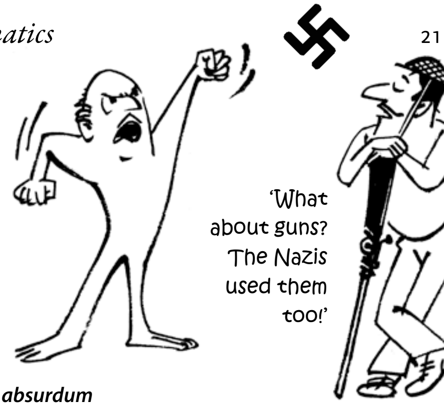
It is customary to consider philosophical theories like mathematical realism, logical positivism, empiricism, intuitionism, and constructivism when studying the philosophy of mathematics. But we shall try to steer clear of these murky depths here.

Nature of Mathematics

Mathematics is a formal and not empirical science. What is a formal science? A formal science endeavours to extract the form from a given piece of deductive argument and to verify the logic on the basis of the validity of form, rather than directly to interpret the content at every step. Thus, a favourite technique to prove the fallacy of an argument is to substitute hypothetical axioms in its form so

that it leads to an obvious absurdity—*reductio ad absurdum*.

Another important distinguishing feature of a formal science such as mathematics is the use of the deductive method in its arguments, unlike empirical sciences such as physics which use the inductive method to arrive at generalizations.

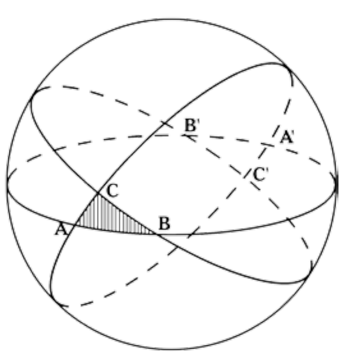


Reductio ad absurdum

Nature of Mathematical Entities

Are mathematical entities real? If they are not real, then whatever name we choose to call them by—*abstract* or *conceptual*—the fact remains that they exist only in our mind, a figment of our imagination—not unlike our feelings, though possibly a bit different.

It is common to acknowledge only the second possibility—that mathematical objects are definitely conceptual entities. But what does the word *conceptual* mean here? *Conceptual*, with respect to mathematical entities, means that they are hypothetical—they may or may not have any correlation with reality. In that case, these entities could be represented and interpreted in any number of ways. This fact has surprising consequences. For example, if numbers are represented by some well-structured sets—as we shall do in the section on number theory—and the operations addition, subtraction, multiplication, and division are redefined for these sets, then the sets themselves may be thought of as numbers without any loss of generality. Yet another example is that of spherical geometry. The lines of the Euclidean plane can be thought of as equatorial circles and points as poles on a spherical surface without any loss of understanding. Only the operations on lines and points will have to be redefined so that Euclidean axioms still hold true.



Spherical geometry: Three equatorial circles forming a triangle ABC; the sum of three angles here is not 180°

But what are the consequences of mathematical entities being conceptual?

On Concepts being Hypothetical

The mathematical universe consists of conceptual objects alone. There is no direct relation between mathematical entities and the phenomenal objects of the empirical world. And these mathematical objects are only indirectly correlated to existent objects and interpreted as such by the human mind. For a given system of mathematical truths, we try to interpret factual truths of the external world in such a way that they fit the mathematical model we have developed. And it may not be possible to match every mathematical model with some external reality. In other words, our mathematical models and external objective reality are connected only by our interpretation of the model. Nevertheless, it is worth noting that there is no one-to-one relationship between these two domains. There can be different mathematical explanations for the same event and, conversely, there can also be different physical interpretations of the same model. This is illustrated in the example below where we try to model real-world addition. Let us define two operators *P* and *Q*, such that we have the following relations:

- 1 *P* 1 *Q* 2
- 1 *P* 2 *Q* 3
- 2 *P* 3 *Q* 5, and so on.

Given the above axioms, the operators *P* and *Q* could be interpreted as *plus* and *equal to* respectively; thus 1 plus 1 equals 2. Are other interpretations of *P* and *Q* possible? Yes. *P* and *Q* may also be interpreted as

equal to and *subtracted from* respectively so that $2\ P\ 3\ Q\ 5$ could be read as 2 equals 3 subtracted from 5. Again, Q could also be interpreted as *greater than or equal to* instead of *equal to*, in which case the above statement would read 2 plus 3 is greater than or equal to 5. In each of these cases we have a reasonable interpretation of the axioms, though the different interpretations of the operators P and Q are not mutually compatible. Thus we see that the same model can be interpreted in three different ways.

I always like comparing this triad of the mathematical model, the objective world, and our interpretation to that of *śabda*, *artha*, and *jñāna*—word, object, and meaning. The mathematical model of the world is equivalent to *śabda*, the world to *artha*, and the interpretation to *jñāna*. This is the way in which mathematical concepts relate to the objects of experience through an interpretation of events that is entirely a product of our thinking.

Mathematics and Physics

Let us now compare the theories of mathematics and physics. What we first notice is that mathematical truths are necessary truths, that is to say, truths deducible from axioms, and true in each and every alternative system (or universe) where the axioms hold. In other words, mathematical truths are true by definition and not incidentally. Immanuel Kant, the celebrated German philosopher, called them *a priori truths*. Empirical truths, on the other hand, are *a posteriori truths*, only incidentally true. All physical facts are, surprisingly, only incidentally true. They may not be true in an alternative world or in an alternative physical system.

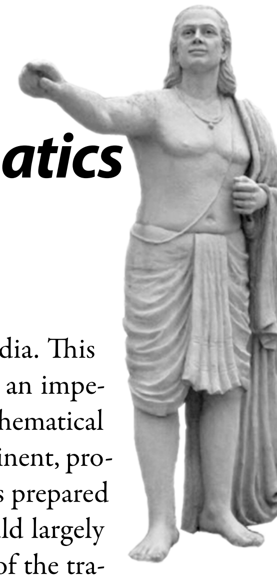
Abstract thinking: Is doing mathematics an exclusively human trait?

For example, take the speed of light. Physicists tell us that the speed of light is a constant, nearly 300,000 km/sec. Now why should the speed of light be this value? Can it not be a different value? Would the physical world appear different if the speed of light were different? When we say that 'The speed of light is nearly 300,000 km/sec' is not a necessarily true statement, then we mean that we can postulate, without fear of any technical objections, another universe where the speed of light is different, say, 310,000 km/sec. Of course, that world would be unlike ours and is not known to exist, but this line of thinking gives us a hint that there is no a priori reason for physical constants to have the immutable values that characterize them—however real they may be for us. In fact, Vedanta boldly proclaimed a long time ago that the physical universe does not have any a priori reason for its existence, and Buddhist thought has also followed this great tradition.

Here it may be of interest to draw a comparison with Nyaya, the traditional Indian system of logic. Nyaya is an empirical philosophy and is fully imbued with realism. Therefore, in its traditional five-step syllogism (*pañcāvayava anumāna*), it is mandatory to cite a real-life example (*dṛṣṭānta*) while drawing an inference from given premises. This step is much like deducing a specific instance from a general principle. And because of this thoroughly realistic approach, postulating a hypothetical universe within Nyaya discourse is virtually impossible, because that would lack real-world examples. In the mathematical domain, on the other hand, every entity is hypothetical, and entities get connected to the real world only through the interpretations applied to them. So we can postulate a hypothesis anytime and anywhere. Though Nyaya too, as a system of formal logic, has its own hypothetical concepts, its grounding in the real world restricts its conceptual flexibility. Hence, Nyaya as a logical system is able to deduce only a subset of the truths which mathematical logic is able to derive. (*To be concluded*)

A Brief History of Indian Mathematics

Prof. Vijaya Kumar Murty



Mathematics has played a significant role in the development of Indian culture for millennia. Mathematical ideas that originated in the Indian subcontinent have had a profound impact on the world. Swami Vivekananda said: 'You know how many sciences had their origin in India. Mathematics began there. You are even today counting 1, 2, 3, etc. to zero, after Sanskrit figures, and you all know that algebra also originated in India.'[1]

It is also a fitting time to review the contributions of Indian mathematicians from ancient times to the present, as in 2010, India will be hosting the International Congress of Mathematicians. This quadrennial meeting brings together mathematicians from around the world to discuss the most significant developments in the subject over the past four years and to get a sense of where the subject is heading in the next four. The idea of holding such a congress at regular intervals actually started at the Columbian Exhibition in Chicago in 1893. This exhibition had sessions to highlight the advancement of knowledge in different fields. One of these was a session on mathematics. Another, perhaps more familiar to readers of *Prabuddha Bharata*, was the famous Parliament of Religions in which Swami Vivekananda first made his public appearance in the West.

Following the Chicago meeting, the first International Congress of Mathematicians took place in Zurich in 1897. It was at the next meeting at Paris in 1900 that Hilbert formulated his now famous 23 problems. Since that time, the congress has been meeting approximately every four years in different cities around the world, and in 2010, the venue will be Hyderabad, India. This is the first time in its more than hundred-year history that the congress will be held in India. This meeting could serve as an impetus and stimulus to mathematical thought in the subcontinent, provided the community is prepared for it. Preparation would largely consist in being aware of the tradition of mathematics in India, from ancient times to modern, and in embracing the potential and possibility of developing this tradition to new heights in the coming millennia.

In ancient times, mathematics was mainly used in an auxiliary or applied role. Thus, mathematical methods were used to solve problems in architecture and construction (as in the public works of the Harappan civilization), in astronomy and astrology (as in the works of the Jain mathematicians), and in the construction of Vedic altars (as in the case of the Shulba Sutras of Baudhayana and his successors). By the sixth or fifth century BCE, mathematics was being studied for its own sake. In modern times, mathematics is studied both for its own sake, as well as for its applications in other fields of knowledge.

The aim of this article is to give a brief review of a few of the outstanding innovations introduced by Indian mathematics from ancient times to modern. As we shall see, there does not seem to have been a time in Indian history when mathematics was not being developed. Recent work has unearthed many manuscripts, and what were previously regarded as dormant periods in Indian mathematics are now known to have been very active. Even a small study of this subject leaves one with a sense of wonder at the depth and breadth of ancient Indian thought.

The picture is not yet complete, and it seems that there is much work to do in the field of the

history of Indian mathematics. The challenges are two-fold. First, there is the task of locating and identifying manuscripts and of translating them into a language that is more familiar to modern scholars. Second, there is the task of interpreting the significance of the work that was done.

Since much of the past work in this area has tended to adopt a Eurocentric perspective and interpretation, it is necessary to take a fresh, objective look. The time is ripe to make a major effort to develop as complete a picture as possible of Indian mathematics. Those who are interested in embarking on such an effort can find much helpful material online.[2]

We may ask what the term 'Indian' means in the context of this discussion. Mostly, it refers to the Indian subcontinent, but for more recent history we include also the diaspora and people whose roots can be traced to the Indian subcontinent, wherever they may be geographically located.

Mathematics in Ancient Times (3000 to 600 BCE)

The Indus valley civilization is considered to have existed around 3000 BCE. Two of its most famous cities, Harappa and Mahenjo-daro, provide evidence that construction of buildings followed a standardized measurement which was decimal in nature. Here, we see mathematical ideas developed for the purpose of construction. This civilization had an advanced brick-making technology (having invented the kiln). Bricks were used in the construction of buildings and embankments for flood control.

The study of astronomy is considered to be even older, and there must have been mathematical theories on which it was based. Even in later times, we find that astronomy motivated considerable mathematical development, especially in the field of trigonometry.

Much has been written about the mathematical constructions that are to be found in Vedic literature. In particular, the *Shatapatha Brahmana*, which is a part of the Shukla Yajur Veda, contains detailed descriptions of the geometric construction of altars for yajnas. Here, the brick-making technology of the Indus valley civilization was put to a new use. As usual, there are different interpretations of the dates of Vedic texts, and in the case of this Brahmana, the range is from 1800 to about 800 BCE. Perhaps it is even older.

Supplementary to the Vedas are the Shulba Sutras. These texts are considered to date from 800 to 200 BCE. Four in number, they are named after their authors: *Baudhayana* (800 BCE), *Manava* (750 BCE), *Apastamba* (600 BCE), and *Katyayana* (200 BCE). The sutras contain the famous theorem commonly attributed to Pythagoras. Some scholars (such as Seidenberg) feel that this theorem is already present in the *Shatapatha Brahmana*. Much later, Bhaskaracharya gave an algebraic proof of this theorem, as opposed to the geometric proof that the Greeks, and possibly the Chinese, were aware of.[3]

The Shulba Sutras introduce the concept of irrational numbers, numbers that are not the ratio of two whole numbers. For example, the square root of 2 is one such number. The sutras give a way of approximating the square root of a number using rational numbers through a recursive procedure which in modern language would be a 'series expansion'. This predates, by far, the European use of Taylor series.

It is interesting that the mathematics of this period seems to have been developed for solving practical geometric problems, especially the construction of religious altars. However, the study of the series expansions for certain functions already hints at the development of an algebraic perspective. In later times, we find a shift towards algebra, with simplification of algebraic formulae and summation of series acting as catalysts for mathematical discovery.

Jain Mathematics (600 BCE to 500 CE)

This is a topic that scholars have started studying only recently. Knowledge of this period of mathematical history is still fragmentary, and it is a fertile area for future scholarly studies. Just as Vedic

philosophy and theology stimulated the development of certain aspects of mathematics, so too did the rise of Jainism. Jain cosmology led to ideas of the infinite. This, in turn, led to the development of the notion of 'orders of infinity' as a mathematical concept. By 'orders of infinity', we mean a theory by which one set could be deemed to be 'more infinite' than another. In modern language, this corresponds to the notion of cardinality. For a finite set, its cardinality is the number of elements it contains. However, we need a more sophisticated notion to measure the size of an infinite set. In Europe, it was not until Cantor's work in the nineteenth century that a proper concept of cardinality was established.

Besides the investigations into infinity, this period saw developments in several other fields such as number theory, geometry, computing with fractions, and combinatorics. In particular, the recursion formula for binomial coefficients and the 'Pascal's triangle' were already known in this period.

As mentioned in the previous section, astronomy had been studied in India since ancient times. This subject is often confused with astrology. Swami Vivekananda has speculated that astrology came to India from the Greeks and that astronomy was borrowed by the Greeks from India. Indirect evidence for this is provided by a text by Yavaneshvara (c. 200 CE) which popularized a Greek astrology text dating back to 120 BCE.

The period 600 BCE to 900 CE coincides with the rise and dominance of Buddhism. In the *Lalitavistara*, a biography of the Buddha which may have been written around the first century CE, there is an incident about Gautama being asked to state the name of large powers of 10, starting with 10^7. He is able to give names to numbers up to 10^{53} (*tallakṣaṇa*). The very fact that such large numbers had names suggests that the mathematicians of the day were comfortable thinking about very large numbers. It is hard to imagine calculating with such numbers without some form of place-value system.

Brahmi Numerals, the Place-value System, and Zero

No account of Indian mathematics would be complete without a discussion of Indian numerals, the place-value system, and the concept of zero. The numerals that we use even today can be traced to the Brahmi numerals that seem to have made their appearance in 300 BCE. But Brahmi numerals were not part of a place-value system. They evolved into the Gupta numerals around 400 CE and subsequently into the Devanagari numerals, which developed slowly between 600 and 1000 CE.

By 600 CE, a place-value decimal system was well in use in India. This means that when a number is written down, each symbol that is used has an absolute value, but also a value relative to its position. For example, the numbers 1 and 5 have a value on their own, but also have a value relative to their position in the number 15. The importance of a place-value system need hardly be emphasized. It would suffice to cite an oft-quoted remark by Laplace: 'It is India that gave us the ingenious method of expressing all numbers by means of ten symbols, each symbol receiving a value of position as well as an absolute value; a profound and important idea which appears so simple to us now that we ignore its true merit. But its very simplicity and the great ease which it has lent to computations put our arithmetic in the first rank of useful inventions; and we shall appreciate the grandeur of the achievement the more when we remember that it escaped the genius of Archimedes and Apollonius, two of the greatest men produced by antiquity.'[4]

A place-value system of numerals was apparently known in other cultures; for example, the Babylonians used a sexagesimal place-value system as early as 1700 BCE, but the Indian system was the first decimal system. Moreover, until 400 BCE, the Babylonian system had an inherent ambiguity as there was no symbol for zero. Thus, it was not a complete place-value system in the way we think of it today.

The elevation of zero to the same status as other numbers involved difficulties that many brilliant mathematicians struggled with. The main problem

was that the rules of arithmetic had to be formulated so as to include zero. While addition, subtraction, and multiplication with zero were mastered, division was a more subtle question. Today, we know that division by zero is not well-defined and so has to be excluded from the rules of arithmetic. But this understanding did not come all at once, and took the combined efforts of many minds. It is interesting to note that it was not until the seventeenth century that zero was being used in Europe, and the path of mathematics from India to Europe is the subject of much historical research.

The Classical Era of Indian Mathematics (500 to 1200 CE)

The most famous names of Indian mathematics belong to what is known as the classical era. This includes Aryabhata I (500 CE), Brahmagupta (700 CE), Bhaskara I (900 CE), Mahavira (900 CE), Aryabhata II (1000 CE), and Bhaskaracharya or Bhaskara II (1200 CE).

During this period, two centers of mathematical research emerged, one at Kusumapura near Pataliputra and the other at Ujjain. Aryabhata I was the dominant figure at Kusumapura and may even have been the founder of the local school. His fundamental work, the *Aryabhatiya*, set the agenda for research in mathematics and astronomy in India for many centuries.

One of Aryabhata's discoveries was a method for solving linear equations of the form $ax + by = c$. Here a, b, and c are whole numbers, and we are seeking values of x and y in whole numbers satisfying the above equation. For example, if $a = 5$, $b = 2$, and $c = 8$, then $x = 8$ and $y = -16$ is a solution. In fact, there are infinitely many solutions:

$$x = 8 - 2m$$
$$y = 5m - 16$$

where m is any whole number, as can easily be verified. Aryabhata devised a general method for solving such equations, and he called it the *kuṭṭaka* (or pulverizer) method. He called it the pulverizer because it proceeded by a series of steps, each of which required the solution of a similar problem, but with smaller numbers. Thus, a, b, and c were 'pulverized' into smaller numbers.

The Euclidean algorithm, which occurs in the *Elements* of Euclid, gives a method to compute the greatest common divisor of two numbers by a sequence of reductions to smaller numbers. As far as I am aware, Euclid does not suggest that this method can be used to solve linear equations of the above sort. Today, it is known that if the algorithm in Euclid is applied in reverse order, then in fact it will yield Aryabhata's method. Unfortunately, the mathematical literature still refers to this as the extended Euclidean algorithm, mainly out of ignorance of Aryabhata's work.

It should be noted that Aryabhata studied the above linear equations because of his interest in astronomy. In modern times, these equations are of interest in computational number theory and are of fundamental importance in cryptography.

Amongst other important contributions of Aryabhata is his approximation of π to four decimal places (3.1416). By comparison, the Greeks were using the weaker approximation 3.1429. Also of importance is Aryabhata's work on trigonometry, including his tables of values of the sine function, as well as algebraic formulae for computing the sine of multiples of an angle.

The other major centre of mathematical learning during this period was Ujjain, which was home to Varahamihira, Brahmagupta, and Bhaskaracharya. The text *Brahma-sphuta-siddhanta* by Brahmagupta, published in 628 CE, dealt with arithmetic involving zero and negative numbers.

As with Aryabhata, Brahmagupta was an astronomer, and much of his work was motivated by problems that arose in astronomy. He gave the famous formula for a solution to the quadratic equation $ax^2 + bx + c = 0$, namely

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

It is not clear whether Brahmagupta gave just this solution or both solutions to this equation. Brahmagupta also studied quadratic equations in

two variables and sought solutions in whole numbers. Such equations were studied only much later in Europe. We shall discuss this topic in more detail in the next section.

This period closes with Bhaskaracharya (1200 CE). In his fundamental work on arithmetic (titled *Lilavati*) he refined the *kuṭṭaka* method of Aryabhata and Brahmagupta. The *Lilavati* is impressive for its originality and diversity of topics.

Until recently, it was a popularly held view that there was no original Indian mathematics before Bhaskaracharya. However, the above discussion shows that his work was the culmination of a series of distinguished mathematicians who came before him. Also, after Bhaskaracharya, there seems to have been a gap of two hundred years before the next recorded work. Perhaps this is another time period about which more research is needed.

The Solution of Pell's equation

In Brahmagupta's work, Pell's equation had already made an appearance. This is the equation that, for a given whole number D, asks for whole numbers x and y satisfying the equation

$$x^2 - Dy^2 = 1.$$

In modern times, it arises in the study of units of quadratic fields and is a topic in the field of algebraic number theory. If D is a whole square (such as 1, 4, 9, and so on), the equation is easy to solve, as it factors into the product

$$(x - my)(x + my) = 1$$

where $D = m^2$. This implies that each factor is $+1$ or -1, and the values of x and y can be determined from that. However, if D is not a square, then it is not even clear that there is a solution. Moreover, if there is a solution, it is not clear how one can determine all solutions. For example, consider the case $D = 2$. Here, $x = 3$ and $y = 2$ gives a solution. But if $D = 61$, then even the smallest solutions are huge [see p. 545].

Brahmagupta discovered a method, which he called *samāsa*, by which, given two solutions of the equation, a third solution could be found. That is, he discovered a composition law on the set of solutions. Brahmagupta's lemma was known one thousand years before it was rediscovered in Europe by Fermat, Legendre, and others.

This method appears now in most standard textbooks and courses in number theory. The name of the equation is a historical accident. The Swiss mathematician Leonhard Euler mistakenly assumed that the English mathematician John Pell was the first to formulate the equation, and began referring to it by this name.

The work of Bhaskaracharya gives an algorithmic approach—which he called the *cakravāla* (cyclic) method—to finding all solutions of this equation. The method depends on computing the continued fraction expansion of the square root of D and using the convergents to give values of x and y. Again, this method can be found in most modern books on number theory, though the contributions of Bhaskaracharya do not seem to be well-known.

Mathematics in South India

We described above the centres at Kusumapara and Ujjain. Both of these cities are in North India. There was also a flourishing tradition of mathematics in South India which we shall discuss in brief in this section.

Mahavira is a mathematician belonging to the ninth century who was most likely from modern-day Karnataka. He studied the problem of cubic and quartic equations and solved them for some families of equations. His work had a significant impact on the development of mathematics in South India. His book *Ganita-sara-sangraha* amplifies the work of Brahmagupta and provides a very useful reference for the state of mathematics in his day. It is not clear what other works he may have published; further research into the extent of his contributions would probably be very fruitful.

Another notable mathematician of South India was Madhava, from Kerala. Madhava belongs to the fourteenth century. He discovered series expansions for some trigonometric functions such as the sine, cosine, and arctangent that were not known in Europe until after Newton. In modern terminology, these expansions are the Taylor series of the func-

On 22 May the King of Norway presented the **Abel Prize for 2007** to the distinguished probabilist **S R Srinivasa Varadhan**. Considered as the Nobel Prize for mathematics, the Abel Prize is awarded by the Norwegian Academy of Science and Letters for outstanding work in mathematics, work of extraordinary depth and influence in the mathematical sciences. Varadhan received the prize for 'his fundamental contributions to probability theory and in particular for creating a unified theory of large deviations'. Kristian Seip, chairman of the Abel Committee, says: 'Varadhan's work has great conceptual strength and ageless beauty. His ideas have been hugely influential and will continue to stimulate further research for a long time.'

tions in question.

Madhava gave an approximation to π of 3.14159265359, which goes far beyond the four decimal places computed by Aryabhata. Madhava deduced his approximation from an infinite series expansion for $\pi/4$ that became known in Europe only several centuries after Madhava (due to the work of Leibniz).

Madhava's work with series expansions suggests that he either discovered elements of the differential calculus or nearly did so. This is worth further analysis. In a work in 1835, Charles Whish suggested that the Kerala school had 'laid the foundation for a complete system of fluxions'. The theory of fluxions is the name given by Newton to what we today call the differential calculus. On the other hand, some scholars have been very dismissive of the contributions of the Kerala school, claiming that it never progressed beyond a few series expansions. In particular, the theory was not developed into a powerful tool as was done by Newton. We note that it was around 1498 that Vasco da Gama arrived in Kerala and the Portuguese occupation began. Judging by evidence at other sites, it is not likely that the Portuguese were interested in either encouraging or preserving the sciences of the region. No doubt, more research is needed to discover where the truth lies.

Madhava spawned a school of mathematics in Kerala, and among his followers may be noted Nilakantha and Jyesthadeva. It is due to the writings of these mathematicians that we know abpout the work of Madhava, as all of Madhava's own writings seem to be lost.

Mathematics in the Modern Age

In more recent times, there have been many important discoveries made by mathematicians of Indian origin. We shall mention the work of three of them: Srinivasa Ramanujan, Harish-Chandra, and Manjul Bhargava.

Ramanujan (1887–1920) is perhaps the most famous of modern Indian mathematicians. Though he produced significant and beautiful results in many aspects of number theory, his most lasting discovery may be the arithmetic theory of modular forms. In an important paper published in 1916, he initiated the study of the τ function. The values of this function are the Fourier coefficients of the unique normalized cusp form of weight 12 for the modular group $SL_2(Z)$. Ramanujan proved some properties of the τ function and conjectured many more. As a result of his work, the modern arithmetic theory of modular forms, which occupies a central place in number theory and algebraic geometry, was developed by Hecke.[5]

Harish-Chandra (1923–83) is perhaps the least known Indian mathematician outside of mathematical circles. He began his career as a physicist, working under Dirac. In his thesis, he worked on the representation theory of the group $SL_2(C)$. This work convinced him that he was really a mathematician, and he spent the remainder of his academic life working on the representation theory of semi-simple groups. For most of that period, he was a professor at the Institute for Advanced Study in Princeton, New Jersey. His *Collected Papers* published in four volumes contain more than 2,000 pages.[6] His style is known as meticulous and thorough and

his published work tends to treat the most general case at the very outset. This is in contrast to many other mathematicians, whose published work tends to evolve through special cases. Interestingly, the work of Harish-Chandra formed the basis of Langlands's theory of automorphic forms, which are a vast generalization of the modular forms considered by Ramanujan.[7]

Manjul Bhargava (b. 1974) discovered a composition law for ternary quadratic forms. In our discussion of Pell's equation, we indicated that Brahmagupta discovered a composition law for the solutions. Identifying a set of importance and discovering an algebraic structure such as a composition law is an important theme in mathematics. Karl Gauss, one of the greatest mathematicians of all time, showed that binary quadratic forms, that is, functions of the form

$$ax^2 + bxy + cy^2$$

where a, b, and c are integers, have such a structure. More precisely, the set of primitive $SL_2(Z)$ orbits of binary quadratic forms of a given discriminant D has the structure of an abelian group. In fact, this is the ideal class group. After this fundamental work of Gauss, there had been no progress for several centuries on discovering such structures in other classes of forms. Manjul Bhargava's stunning work in his doctoral thesis, published as several papers in the *Annals of Mathematics*, shows how to address this question for cubic (and other higher degree) binary and ternary forms.[8] The work of Bhargava, who is currently Professor of Mathematics at Princeton University, is deep, beautiful, and largely unexpected. It has many important ramifications and will likely form a theme of mathematical study at least for the coming decades. It is also sure to be a topic of discussion at the 2010 International Congress of Mathematicians in Hyderabad.

G RAMAKRISHNA RAO

Ramanujan (left) and Harish-Chandra

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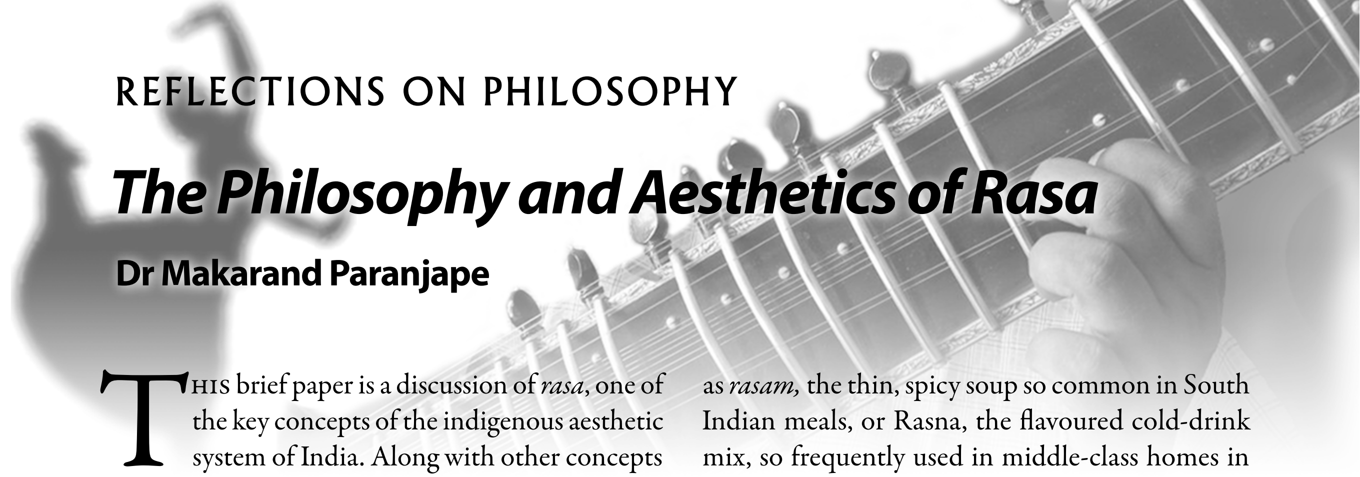

The Philosophy and Aesthetics of Rasa

Dr Makarand Paranjape

This brief paper is a discussion of *rasa*, one of the key concepts of the indigenous aesthetic system of India. Along with other concepts such as *dhvani* (implied meaning), *alaṅkāra* (rhetoric, figure of speech), *vakrokti* (evocation, insinuation, sarcasm), and *aucitya* (congruity, aptness), it helps to constitute a comprehensive aesthetic philosophy which has informed Indian civilization from the earliest times to the present. Despite the impact of Western ideas on the Indian psyche, rasa aesthetics continues to underlie our artistic and visual sensibilities. One proof of this is its prevalence in the most popular and powerful of our artistic media, Bollywood cinema. I would argue that ninety per cent of all Mumbai films are dominated either by *śṛṅgāra* (romantic) or *vīra* (heroic) rasas; and also, that this is primarily a cinema of emotion rather than action or intellect. We go to Bollywood movies to laugh and cry, to feel pain and pleasure, to escape from our daily routine, but also to engage with our realities through exaggeration, fantasy, and romance. In our films—like the older folk forms of performance—dance, song, and music are an integral part of the experience of viewing. This distinct mode of performance, thus, harks back to Bharata's *Natyashastra* (dated variously between 2nd cent. BCE and 4th cent. CE). Unfortunately, most Indians are totally ignorant of this rich aesthetic tradition. This essay on *rasa* is thus an attempt to glimpse the deeper structures and values that underlie Indian aesthetic thinking.

I have already tried to show how relevant rasa continues to be by citing the example of Mumbai cinema. While I was writing this essay, I found another startling reminder of how rasa plays such an important role in our day-to-day life. No, I am not referring to the popular derivatives of the word such as *rasam,* the thin, spicy soup so common in South Indian meals, or Rasna, the flavoured cold-drink mix, so frequently used in middle-class homes in summers. Nor am I referring to Indian cuisine, in which rasa is so important. All these examples no doubt link food and aesthetic sensibility, showing how some of the defining features of our way of life endure despite rapid changes and modernization. My reference is actually to the important role that rasa plays in the celebration of all our festivals. I will consider the example of Holi, the festival of colours; Holi, because my paper is being written during this period.

Relishing Rasa

One can hardly choose a better occasion to discuss rasa than Holi, which is observed each year on Phalgun Purnima, a full-moon night in spring. This ancient spring festival celebrated all over India is marked by the burning of all impurities in an effigy, as depicted in the story of Prahlad and Holika, before one welcomes one's own and others' renewal with a frenzy of colour, song, dance, and, oftentimes, intoxication. Holi is the rite of victory of spring over winter, of Eros over Thanatos, depicted in the frolics of Krishna with his playmates, the gopis of Vrindavan. Interestingly, the Shaivite legend associated with Holi has to do with the burning of Kamadeva when he tried to disturb Shiva's tapas. After Rati, his wife, pleads for her husband's life, Kama is restored, but only in a mental form; hence his other name is Ananga or the bodiless one. Kama, or desire, now becomes an abstraction, something that resides in the minds of all of us. Holi is also a carnival of transgression and subversion, erasing class and gender differences, upturning hierarchies, creating playful disorder, and above

all, lots of fun and mischief. In such a celebration, what is of paramount importance is the rasa of Holi, that is, the relish of the festival. All the colours, fun, frolic, eating, and drinking lead to the tasting of the rasa of Holi. Similarly must every festival in India—whether it is Durga Puja or Ganesh Chaturthi or Pongal or Diwali—be enjoyed. Without this sense of participatory delight or rasa, the festival is drab or uninteresting.

Rasa, therefore, informs our daily life as it does our aesthetic pursuits. But what is rasa? A simple definition would be 'juice'. Rasa is actually the juice, the sap, and the lusty-tasty meaning of life that we so hunger and crave to enjoy. Rasa, in all its various meanings, is thus akin to 'juice', which too is so rich in connotation and denotation. We all want to extract, as it were, the juice of life, or to use the word as a verb, to juice every experience, including the very reading of this essay. From this perspective, rasa has much to communicate to us.

Rasa is central not only to an understanding of the entire spectrum of the arts and crafts of India, but also to any understanding of India as a whole, especially of the lived reality of this complex and fascinating country. The word makes its first appearance in the Rig Veda (c. 1700 BCE), the oldest Indian text, and one of the oldest known texts of humankind. Here it simply means juice, particularly of the magical-medicinal plant, Soma—the highly inspiring, symbolically charged elixir of the legendary plant. Agni and Soma, of course, are like Yin and Yang, the male and female principles, which must be in balance. Agni heats while Soma cools; the Sun is Agni while the Moon is Soma. One fires up, the other stills. These are not just metaphors, but spiritual principles.

Some have considered the Soma-rasa to be simply a hallucinogenic drug, a narcotic, an extract of a plant that could intoxicate those who consumed it. From that point of view, our progenitors, the great rishis, were often 'intoxicated'. But was this 'high' they spoke of only a derivate of some material, mind-altering substance? Or did it refer to spiritual power, afflatus, the ecstasies of higher consciousness? The chanting of the Vedas in the proper manner was supposed to induce this heightened state, for instance. That is why the Vedas were performative, not scriptural. They were not literature or simply texts, but chants, mantras, songs. By singing them properly one could subdue the elements, overcome one's enemies, fulfil one's desires, cure illnesses, and so on. The world, in other words, was linked to the word. The latter produced the former. Or to put it differently, the word was the juice of the world. The material, the spiritual, and the aesthetic were thus fused into a synergy. To cognize the world, to act on it, to alter it—all these were actually ways of enjoying the world. The object of knowledge was also the object of delight. The kinaesthetic and the aesthetic went hand in hand.

In the Upanishads, especially in the *Taittiriya*, rasa begins to acquire a more complex character. Here it means not only juice, but spiritual essence and flavour, and furthermore, as something that moves or transforms the mind. The *Taittiriya Upanishad* has the famous declaration: '*Raso vai saḥ*; He (or That) is the flavour of all existence'—by tasting or grasping which, our lives can become blessed. These examples from the Vedas show that the idea of rasa is as old as systematic Indian thinking.

Rasa in Drama: Knowing and Enjoying

As most major ideas migrated from the Vedic core to the other domains, so did rasa. In fact, Bharata made it central to his treatise on the arts, the *Natyashastra*. *Nāṭya*, the art and science of drama more specifically, but performance more generally, was itself the fifth Veda because, like the Vedas, it was conducive to the *puruṣārtha*s, the cardinal aims of life—namely dharma, righteousness, artha, wealth and power, kama, desire and sex, and moksha, emancipation or liberation. Because the Vedas were not available to all, a new form of instruction that would appeal to all was required.

Central to this new art form was rasa. As Bharata says, there is no *nāṭya* or performance without rasa:

> There is no natya without rasa. Rasa is the cumulative result of *vibhava* [stimulus], *anubhava* [involuntary reaction], and *vyabhicari bhava* [voluntary reaction]. For example, just as when various condiments and sauces and herbs and other materials are mixed, a taste is experienced, or when the mixing of materials like molasses with other materials produces six kinds of taste, so also with the different *bhavas* [emotions] the *sthayi bhava* [permanent emotions experienced 'inside'] becomes a rasa.
>
> But what is this thing called rasa? Here is the reply. Because it is enjoyably tasted, it is called rasa. How does the enjoyment come? Persons who eat prepared food mixed with different condiments and sauces, if they are sensitive, enjoy the different tastes and then feel pleasure; likewise, sensitive spectators, after enjoying the various emotions expressed by the actors through words, gestures, and feelings, feel pleasure. This feeling by the spectators is here explained as the rasa of natya.[1]

What is interesting is that nowhere does Bharata define *rasa*; he simply says that it is something to be experienced, savoured.

After Bharata, rasa became the basis of a whole aesthetics, as Richard Schechner explains in his brilliant essay 'Rasaesthetics':

> Rasa is flavor, taste, the sensation one gets when food is perceived, brought within reach, touched, taken into the mouth, chewed, mixed, savored, and swallowed. The eyes and ears perceive the food on its way—the presentation of the dishes, the sizzling. At the same time, or very shortly after, the nose gets involved. The mouth waters in anticipation. Smell and taste dissolve into each other. The hands convey the food to the mouth. …
>
> Rasa also means 'juice', the stuff that conveys the flavor, the medium of tasting. The juices of eating originate both in the food and from the body. Saliva not only moistens food, it distributes flavors. Rasa is sensuous, proximate, experiential. Rasa is aromatic. Rasa fills space, joining the outside to the inside. Food is actively taken into the body, becomes part of the body, works from the inside. What was outside is transformed into what was inside.[2]

One way to understand the salient features of rasa as experienced through *nāṭya* is to contrast it with the Greek idea of theatre. As Schechner argues, 'An aesthetic founded on rasa is fundamentally different than one founded on the "theatron", the rationally ordered, analytically distanced panoptic' (29). In other words, in the European tradition, knowing is through seeing—theorema, theorem, theory, theatre, all linked to seeing, looking, gaze, all of which require distance and separation, while rasa is more immediate and experiential. As one of the foremost commentators on the *Natyashastra*, the great Tantric philosopher and mystic of the eleventh century, Abhinavagupta, put it, 'Rather than being objectively known, it is to be understood through savouring.'[3] If we apply such a definition, somewhat self-reflexively, to this very discussion of rasa, we will understand the concept of rasa only by knowing and enjoying how it is being presented to us *right now*. In other words, what the concept of rasa challenges us to do at the outset is to forge a causal and necessary connection between knowing and enjoying.

Rasa as Cognition and as Puruṣārtha

Bharata states this categorically when he declares: 'Without rasa, there is no cognition.'[4] When I first came across this line, it certainly opened a door of perception. I never tire of repeating it to my students almost as a pedagogical imperative. If you're not having fun, you're not learning. That's the test. How can we be sure? Because to learn is a basic human drive, as basic as ingestion, locomotion, or reproduction. Cognition or the constitution of meaning has to be accompanied by a certain enjoyment. Conversely, we cannot learn what we dislike or find un-engaging, uninteresting, or tedious. Rasa, then, even if it suggests taste and savour, is actually a cognitive category.

Bharata identifies eight rasas in the famous sixth chapter of the *Natyashastra*: *śṛṅgāra* (erotic, effulgent), *hāsya* (humorous, happy), *karuṇa* (compassionate, pitiful), *raudra* (angry or inflamed), *vīra* (heroic, energetic), *bhayānaka* (terrifying, fearful),

bībhatsa (disgusting, odious), and *adbhuta* (wondrous, enchanting). Each rasa has a corresponding dominant or stable mental state, or *sthāyi-bhāva*, which helps to produce it: *rati* (love), *hāsa* (mirth), *śoka* (sorrow), *krodha* (anger), *utsāha* (enthusiasm), *bhaya* (fear), *jugupsā* (disgust), and *vismaya* (wonder) respectively. *Bhāva*s, of course, are not just stable emotional states, but states of being. Rasa aesthetics is thus based on an understanding and conceptualization of the structure of human experience. It is more fruitful to consider it descriptive rather than prescriptive. Bharata also defines how the various rasas are generated or produced in his famous 'rasa sutra': '*vibhāvānubhāva-vyabhicāri-saṁyogāt rasa niṣpattiḥ*; the combination of *vibhāva*s (determinants) and *anubhāva*s (consequents) together with *vyabhicāri-bhāva*s (contributory or transitory states) produces rasa'.

These eight rasas, according to Bharata, actually fall into two groups of four each: *śṛṅgāra*, *hāsya*, *vīra*, and *adhbuta* on the one hand, and *karuṇa*, *raudra*, *bhayānaka*, and *bībhatsa* on the other. If we examine their *sthāyi-bhāva*s, we will see that the first group is pleasant, while the second is unpleasant. While we all like to experience *rati*, *hāsya*, *utsāha*, and *vismaya*, we try to avoid *śoka*, *krodha*, *bhaya*, and *jugupsā*. The rasas promote the *puruṣārtha*s by pointing out the consequences of our actions; some actions will have pleasant consequences, while others will have deeply disturbing ones. The plot of a play will show which actions are conducive to a good or virtuous life and which are not. The abduction of Sita, for instance, will induce *bhaya*, *śoka*, *jugupsā*, or even *krodha*—all these emotions suggest how immoral or terrible an act it was—and the corresponding rasas—*bhayānaka*, *karuṇa*, *bībhatsa*, or *raudra* will be experienced by the spectators of the performance. The consequences of Ravana's act, eventually, include not just the death of Ravana, the perpetrator of the deed, but the destruction of his whole city, along with many of its inhabitants. There is thus an intrinsic relationship between the rasas and the *puruṣārtha*s.

In the subsequent literature, there has been much debate on the number of rasas, their relationship to the *bhāva*s, and on how they are produced in a work of art and perceived by the audience. In tracing the development of the concept of rasa from Bharata to present times, we come across the names of several commentators, notably Bhatta Lollata and Shankuka (8th cent.), Bhatta Nayaka (10th cent.), and most importantly, Abhinavagupta (11th cent.). The latter added a ninth rasa, the *śānta*, or tranquil, which he considered to be the substratum of all the other rasas. According to Abhinavagupta, the appreciation of a work of art leads us to a state of joyous satisfaction that comes from the full savour of the performance. Just as a yogi reaches such a state of detached enjoyment of the world through a proper understanding of it, an auditor arrives at a similar state of blissful repose after fully juicing an aesthetically superior performance. Regarding the *sthāyi-bhāva* of the *śānta rasa,* Abhinavagupta considers it to be *tattvajñāna*, categorical knowledge or knowledge of the absolute Reality. Because *tattvajñāna* leads to moksha, it is the appropriate *sthāyi-bhāva* of the *śānta rasa*. Like other Kashmir Shaivites, Abhinavagupta considers epistemology to be the key to self-knowledge; he thus makes the *sthāyi-bhāva* of *śānta rasa* also an epistemological category. In Prof. Kapil Kapoor's rendering: '*Tattvajnana* is the means of *moksha*, and therefore it should be accepted as the *sthayibhava* of *shanta rasa*. *Tattvajnana* is the name for *atmajnana* (self-knowledge or knowledge of the self). … This knowledge of the self, that is, *atmasakshatkara* or *tattvajnana*, is the *sthayibhava* of *shanta rasa*. … So, *atma,* the self, endowed with unalloyed properties … takes the form of the *sthayibhava* for *shanta rasa*.'[5] This is how the great acharya Abhinavagupta traces the source of the *mahārasa śānta* to our very own self, our own true nature or *svarūpa*.

Later theorists, especially those belonging to the Vaishnava devotional school of Chaitanya Mahaprabhu, considered *bhakti* or fervent devotion to be the *mahārasa* or greatest rasa. In a sense, bhakti was not a new rasa but a version of *śṛṅgāra*, the principal rasa as already identified by Bharata and

reiterated by Abhinavagupta. Using the Bhagavata as their key text, these Vaishnavas offered a very sophisticated analysis and experience of *bhakti rasa* in their *kathā*s or performative narratives. Some of these are popular to this day.

In Crowds and in the Gut

One of the issues in rasa aesthetics concerns the manner in which the art-emotion, or feeling, is transmitted and then generalized. For a full appreciation of rasa, the reader or auditor must be fully in tune with the author or dramatist, almost becoming his cohort, or should I say, co-heart (*sahṛdaya*) in the process. Another important link in the theory was the idea of *sādhāraṇīkaraṇa*, or the making common of the rasa, the process by which the art-emotion gets universalized in such a manner that not just individuals, but the whole audience experiences a similar joy of relish. This fusing of a large group of people into one ecstatic mob can be seen at any good rock concert. Several theorists and commentators including Abhinavagupta likened the aesthetic experience to the spiritual one. Aesthetic pleasure was akin to spiritual bliss: *brahmānanda-sahodara*; both were born, as it were, of the same womb. That is why the highest aesthetic experience was also accompanied by a sense of *camatkāra* or epiphany.

The whole rasa aesthetics, as we have seen, is oral and gustatory rather than visual or optical. As Schechner puts it, ‘Fundamentally, the attainment of pleasure and satisfaction in a rasic performance is oral … and the satisfaction is visceral, in the belly.’[6] This is as it should be in a culture which was both pagan and sensuous but also highly sophisticated and reflective. A performance was a common part of a feast, usually a religious feast, as is the case with Holi. Schechner, using neurobiology, contends that rasa acts on our gut, that is, on the enteric nervous system: ‘The gut—esophagus, stomach, intestines, and bowel—has its own nervous system. This system does not replace or pre-empt the brain. Rather it operates alongside the brain, or—evolutionarily speaking—“before” or “underneath”—the brain’ (36). While I would not go so far as to suggest that rasa is some kind of primitive art experience, I would argue that it is more integral and satisfying because it involves our whole being—physical, emotional, mental, and spiritual. What is more, the rasa aesthetic is participatory, with the audience appreciating and vocally applauding performers.

Recovering the Aesthetic Tradition

I would like to end by mentioning an important feature of Indian philosophical traditions, which also applies to our present discussion of rasa. According to the *paramparā*, there must be a causal relationship between three essential features of any given philosophical structure: *anubhava*, *ācāra*, and *vicāra*, or experience, conduct, and thought. We consider the Vedas as the bedrock of our civilization because they are an embodiment or record of the most profound experiences of the Divine in all its facets. The rishis experienced a cosmos populated with gods and great energies. This cosmos was not alien and separate from those who perceived it but very much a part of themselves. They ‘heard’ the mantras which they then sang and committed to memory. These mantras, in turn, ‘produced’ the gods and gave rise to the various worlds. Without the experience quotient of the Vedas, the later systems of phi-

losophy would be hollow. Similarly, in modern times, our tradition has been renewed and revitalized by great mystics and masters like Sri Ramakrishna. They experienced the highest truths and realities. It is upon the certitude of their *anubhava* that complex philosophical systems rest. Of what avail are thought systems and philosophies to those who have no spiritual experience? Are they not empty words, devoid of substance or backing? Similarly, the great mystics, gurus, and sages demonstrated in their lives that their conduct was in keeping with their experiences. Again, Sri Ramakrishna may serve as a great example. In his daily life and conduct, he lived the great truths of the Indian tradition. Finally—and not all sages and mystics bothered to do so—there was an intellectual articulation of his experiences. In the Indian tradition, any contradiction between thought and deed or precept and practice is considered a sign of imperfection, a failing. A guru who does not practise what he or she preaches is considered not worthy of being followed or worshipped.

It seems to me that the idea of rasa has similarly to be experienced, demonstrated in one's daily life, and only then expounded as a philosophical or theoretical model. We must experience the rasa of life, experience the highest ananda or bliss while being embodied, live and act as if this rasa and ananda inform our existence, and only then speak of the theory of rasa as an aesthetic philosophy. This is the ideal. But more practically, our civilization expected and created an ambience where the flow of rasa was possible and appreciated. Nowadays, I am afraid that this high aesthetic sensibility seems to have vanished from our midst. Modern, especially urban, life seems to be marked by an unprecedented ugliness and absence of rasa: ugly buildings, ugly homes, ugly vehicles, ugly pots and pans, tasteless food, and an absence of refinement in popular taste. What is heartening, however, is that in the midst of utter squalor and wretchedness, one sometimes comes across a beautiful *rangoli* or decorative pattern outside the poorest of our hovels. Suddenly, in the midst of *bībhatsa* or disgust, one experiences the *camatkāra* of the *adhbhuta rasa*.

Now that India is gradually regaining its prosperity as a nation, I hope that the systems and products we design, whether for the now or the future, may be full of rasa. They must have juice in them. They should be full of nutrition, both literally and metaphorically. They should be able to make the consumer both participate in their essence and experience a sense of satisfaction. They should be flavourful and joyful. They should not alienate the maker and the user, but integrate and connect both of them. They should, in a word, be so designed as to be tasted, relished, enjoyed, and experienced. PB

***Eminent Kathak exponents Raghav Raj and Mangala Bhat give expression to the full range of nine* bhāvas**

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4. Bharata, *Natyashastra*, 6.32.
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Philosophy of Language in the Vaiyakarana Tradition

Dr Kapil Kapoor

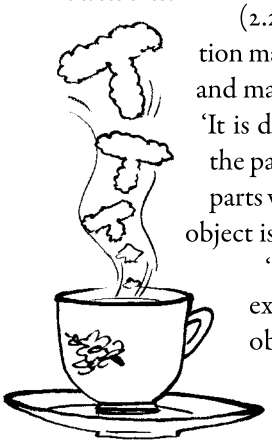
(*Continued from the previous issue*)

Limits of Language

The total reality of an object is not expressible in words because of (i) the process of imposition of mental concepts on the object; (ii) the intrinsic limitations on the potential of words to denote objects; and (iii) the dependence of verbal usage on the speaker's perception and the hearer's reception, governed by their experience. We have already noted the process of imposition of a subjective construct on the object in the process of word use. Consider now the intrinsic limitations on the denotative potential of language (*abhidhā*). Verbal knowledge is assymetrical with the objects talked about because imaginative reconstruction is involved in verbal cognition. So, how valid is the knowledge generated by words? In fact, even visual perception is not reliable and may not give correct or complete knowledge. For instance, 'The sky is perceived as a surface and the firefly as fire. There is no surface in the sky nor is the firefly fire.'[23] Again, 'The perception of (real) water and of such things as a mirage is the same. In spite of the similarity of perception, a mirage is not water' (2.287). Moreover, visual perception may give only partial knowledge and may not cover the whole object: 'It is difficult for anybody to see all the parts of an object. From the few parts which are perceived, the whole object is inferred' (2.161).

'This being so, the wise must examine by reasoning even an object apprehended by direct perception' (2.141). Intellection is clearly necessary to interpret, to cognize accurately, even a perceived object.

The taste of tea



CHANDRA

A word does not express as its meaning everything that exists in, or characterizes, an object. For example, a word does not express the colour or form associated with the object. 'The pot broke.' Was 'the pot' brown or black? Again, qualities like 'whiteness' are not meant to be conveyed by the noun 'rice' (2.69). But the noun 'milk' invariably conveys the quality 'whiteness', though its 'richness' or 'purity' is not implicitly conveyed and will have to be articulated independently. However, there are always inherent limits, any number of them, to such implicit connotation. For example, 'taste' is not meant to be implicitly conveyed by such nouns as 'tomato', 'tea', or 'coffee'.

The meaning of a word is a general concept, a concept that amounts to saying 'something exists' (2.119). No particular form or shape is part of the meaning. When words like *apūrva* (remote consequences of an act), *devatā* (deity), and *svarga* (heaven) are used, we comprehend no particular or definite shapes or forms. What about *aśva* (horse), or *gau* (cow), however? We do visualize or associate some general forms with these terms. But this perception of form in response to certain words is the result of memory—we recall the form of the object that is repeatedly associated with the word. Bhartrihari says, the external forms conveyed by some words (*ghaṭa*, pot, *paṭa*, cloth, etc.) are based upon distinct reminiscences, residual traces of the actual experience of the corresponding external object possessing a shape. Other words (such as *apūrva* and *svarga*) have a meaning in the nature of bare under-

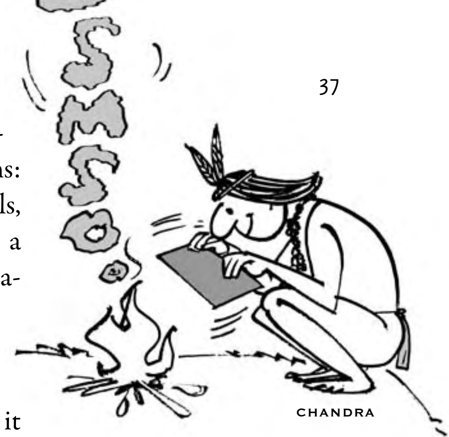
standing not characterized by any shape (2.133). So the perception of form does not come within the range of effects of words; it is instead the result of a special effort, the experience of the repeated use of a particular word for a corresponding object (2.120).

Even a word like *ghaṭa* does not denote a definite shape or form of a pot, but only the general idea common to all jars, or to the jars we have known, and not all the possible forms and shapes that jars can possible have (2.123). Hence the nebulosity or instability of meaning—each one of us brings his or her memory to the act of constituting an object. We all have our separate, particular pot, which others do not, or cannot, see, and which we cannot possibly make others see; it is a strictly private vision, *ātmasākṣātkāra*. What happens is that the speaker has a meaning in mind which is externalized by being rested on an object. One may succeed in this process only to a degree. And the listener also may understand only to a degree. In verbal usage, says Bhartrihari, there is another being, a secondary being that is different from the external, perceptible being.

Extended Meaning

As against this intrinsic partial denotation, there is excess denotation, *saṁsarga-artha*, in that words evoke associated, secondary, and figurative meanings besides the primary connotation: 'Cognitions of hearers are qualified by all associated things (meanings)' (3.14.473).

Bhartrihari gives the example of the word *brāhmaṇa*, which is invariably associated with certain acts and which is understood, through convention, to possess certain qualities (3.14.481). Similarly the term *putra* (son) is qualified by actions, such as obedience, that are commonly associated with it (3.14.508). And the adjective 'black' may be applied to an object when only a part of it is black (3.14.406). In the same way, words also have, or acquire, secondary or extended meanings on the basis of cognizable resemblances. The use of the word *parvata* for a reflection of a hill in water and *aśva* for a clay horse are cases in point. Bhartrihari explains



Smoke SMS

the basis of such extended connotations: 'Just as a lamp reveals, in an object like a jar, through association (or proximity) things other than that for the illumination of which it was used, in the same way, a word conveys meanings that are different from the one which it is used to convey.' (2.298–9)

Sometimes the meanings so evoked are unintended: 'Though the churning of ignition sticks (*araṇi*) is meant for producing fire, they also produce unintended smoke. In the same way, a word, though used to convey a particular meaning, denotes by association unintended meanings too. Just as (while taking a thing) one cannot abandon something very closely connected to it, in the same way a word cannot but denote what is intimately connected with its primary meaning' (2.300–2).

Consider next the figurative meanings evoked by words. In whole utterances the object referred to may actually stand for another, more general concept. 'When a boy is told, "save the butter from the crows", he does not refrain from protecting it from dogs etc., knowing that the order refers to destructive agents in general' (2.312). 'When an order for feeding somebody is given, the washing of the dishes and plates, though not actually mentioned, is also understood as a part of the act of feeding' (2.318).

When the boundaries of denotation are so fluid, the focus on some one aspect of meaning is brought about by the speaker's *vivakṣā*, intention born of a specific perception, and the hearer, in order to grasp the intended meaning, has to reconstruct the speaker's perception. The same object may be presented differently by different speakers or even by the same speaker on two different occasions. Thus one may use the word 'horse' to denote its endurance or its speed or the beauty of its form. In the same way, the speaker may 'reorder' reality in his or her represen-

tation or expression of that reality: 'Sometimes objects which are far from one another are presented as being connected and those which are near one another are presented as being apart. [There also occurs] separation of what is united and union of what is separated; what are many are represented as one and what is one as many' (2.431–2).

Fluidity of Meaning

There is thus great variety and lack of fixity in the way objects are expressed by words. The word 'water' may refer to a 'drop' or an 'ocean' (2.156), depending on what 'is fashioned by the mind' (3.3.88). In fact, when a word is used, 'a remembrance resembling the experience of the object' figures in the mind of the speaker and also takes place in the hearer's mind (2.417).

Since each speaker and each hearer brings his or her own 'remembrance', there is great relativity in verbal denotation both in comprehension and expression. This has been stated by Bhartrihari through four *kārika*s: 'Just as our senses perceive the same object in different ways, in the same way, an object is understood from words in different ways' (2.134); 'The meaning of words, intended by the speaker to be one thing, is understood by different listeners differently, according to their own background' (2.135); 'With regard to the same things, one's views undergo change. The same person sees the same thing differently at different times' (2.136); 'To one and the same word are attributed many meanings by one and the same person, or by many persons, according to varied circumstances' (2.137).

Consider the following sentences: (i) 'You are like yourself' (where a person's appearance is compared with that at another time or place); (ii) 'Wearing a brown coat and a grey cap, that is Ramesh' (where Ramesh the person is compared to Ramesh who figures in the mind, thus establishing his identity). Even in these descriptions based on similarity and difference, the standard and the object of comparison are based on what figures in the mind. Both similarity and difference 'depend upon the mind' (2.14.567). 'The object is understood as agreeing with the image produced in the mind' (3.14.569). 'Operations based on difference are understood through difference made by the mind. All meanings of words seem to be created by the intention of the speaker' (3.14.570). 'Even real difference,' says Bhartrihari, 'depends on the mind' (3.14.567). Even totally different objects may be cognized as the same' (3.14. 572); this is the basis of metaphors such as '*agninā siñcati*, irrigates with fire'.

Whole sentences too are propositional images, and the role of mental mediation in the determination of their meaning is equally evident. They have to be interpreted in light of a large number of contextual factors, both internal and external (see note 6). Thus, 'an enlightened hearer knows that praise and blame, meant to promote action and abstention from action respectively, are actually unreal. Even if at times the truth is told in the form of praise or blame, the object is always to teach action or abstention from action' (2.319, 324).

In the case of single-word sentences, the process becomes very clear. When someone utters the word 'door', depending on the context, one understands either 'shut' or 'give way' (2.330, 333). Conversely, the process of establishing the meaning of words on the basis of the sentence meaning involves inferential reasoning. For example, the statements '*fetch* a cuckoo' and '*fetch* firewood from the forest' give rise to different mental images of 'fetching'. 'It is hot in here' can be understood to mean, depending on the intention and context, either 'open the windows' or 'put on the fan'. At a more subtle level, 'the sun has set' as spoken by a thief, a worshipper, and a lover will carry altogether different connotations. In sum, language meaning is intrinsically non-literal, *ālaṅkārika*, figurative.

The Realm of Pure Feeling

Literary language is all the more figural—therein images are pervasively and consciously employed to constitute meaning. This phantasm of literary representation is required so that 'the realm of pure feeling can find utterance',[24] or else, 'what could we

know of the innumerable nuances in the aspects of things?' The verbal image is the way in which the ideational content of an object is apprehended and expressed. Taking Keats' 'Ode to a Nightingale' as an example, one can cite a number of images that are shifted analogues of what they represent, conceptual constructs to connote the nuances and the fullness of experience. The Indian thinkers call it *bhāva-abhivyakti*, expression of the *tattva*, the essence or core. From these shifted representations, one is able to construct the 'truth' of the represented object or experience. Keats speaks of a piece of land that is 'melodious' and of a kind of mirth that is 'sunburnt'. And what kind of going away is the 'fade away' of Keats? What are 'murmurous haunts' and what the 'verdurous gloom'? How is a bird 'immortal'? And what is 'embalmed darkness'? As we work through these images, we awaken to newer shades of experience. It is in this manner that the changing intensity of the sunshine in the bubbles makes them 'wink' and the cool, clear wine becomes 'dewy'.

One may also note the juxtaposition and foregrounding of certain images to create patterns or densities of meaning—the repetition of *sad*, *fade*, and *forlorn* and the idea of 'darkness' foreground the central heartache, and the recurrent opposition of 'darkness' and 'light' underscores the toggling of joys and sorrows—the tragedy of course is that the 'light' merely intersperses the pervasive gloom, and that too through divine grace. Read as such, the poem is a highly inflected statement of a complex state of the self that has been captured in subtle verbal images.

Verbal Intuition and Secondary Being

The verbal images that construct objects are centred in the individual consciousness. The listener (or reader), once he or she has understood the meaning of the words separately, has 'a flash of understanding' which is termed the meaning of the sentence. This individual amalgamation brought about by the meaning is indefinable (*avicārita*).[25] Though it cannot be 'explained to others as such and such, it is experienced by everyone within oneself; (but) even the subject (of the experience) is not able to render an account of it to himself' (2.144). A specific faculty is involved in this flash of understanding, a creative faculty. It is *pratibhā*; and all living beings have this: 'Who transforms the voice of the male cuckoo in spring? Who teaches living beings to build nests?' (2.149). Bhartrihari is indifferent to the question whether this faculty is inherited or acquired. But it is obvious that as our intuitions about life deepen, the meanings we grasp become richer and more true to life—our cognitions and responses become more vibrant and varied as our *citta* grows in experience,[26] for 'the consciousness that is projected on the insentient intellect (*acetana citta*) as a reflection (*pratibimba*), is the basis of the use of words'.[27]

In verbal usage, it is the reflection or 'secondary being'—the image that is constructed, predicated, and grasped—that is involved (*vṛtti* on 3.3.39). 'When words convey objects [as meaning], the things so conveyed have a being distinct from their external being. It consists in their figuring in the mind. … Through this being, things are presented as past or yet to come.' This involvement of secondary being is exemplified in negation as well—'not happy' is the reflex of a unitary, positive, secondary being in the speaker's mind. The existence of primary objects is not denied; all that is being claimed is that in language, objects, both existent and non-existent, are talked about in terms of their secondary being. The objects denoted by words are conceived in the mind, and the mind can conceive of objects which have no external being; words then convey a purely mental conception, such as, 'fire-circle' (*alāta-cakra*) and 'hare-horns' (*śaśa-viṣāṇa*).

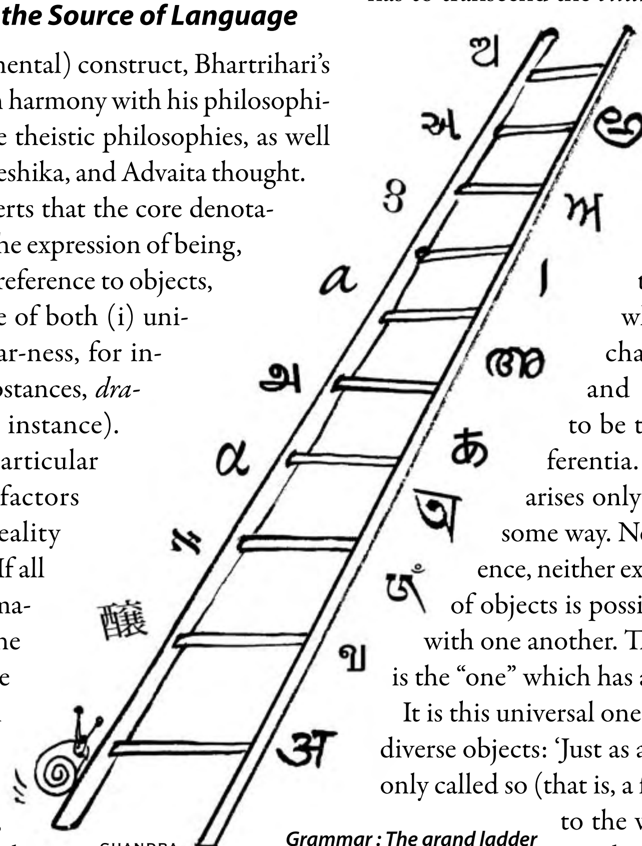
Bhartrhari expatiates on this idea of secondary being, *pravartaka-sattā*, in several *kārika*s: 'Both cognition and words are based on forms existing (in the mind). One does not say "it does not exist" without a basis, and the non-existent does not really differ from the existent in so far as both are mental constructions' (3.7.109). 'All wordly usage,' Bhartrihari says, 'is carried out with mentally-constructed

objects' (3.3.82). 'The primary object has no form till it is determined by the mind' (3.14.350).

It is in this sense of creating the consciousness-centred cognition of *ghaṭa-paṭa-bheda* (the difference between a jar and a piece of cloth) that 'language cuts forms in the ocean of reality'.

Śabda-brahman: the Source of Language

Being a linguistic (mental) construct, Bhartrihari's theory of reality is in harmony with his philosophical grounding in the theistic philosophies, as well as in Sankhya, Vaisheshika, and Advaita thought. For example, he asserts that the core denotation of any word is the expression of being, *sattā*, and that with reference to objects, words are expressive of both (i) universals (*ghaṭatva*, jar-ness, for instance), and (ii) substances, *dravya* (*ghaṭa*, jar, for instance). It is through the particular forms as limiting factors that the ultimate reality (*sattā*) is expressed. If all objects are transformations (*vivarta*) of one ultimate Reality (the *sat* of Advaita), then each object has a universal aspect and a particular aspect, and both are denoted by words. In this conceptualization of the reality of objects and of how valid knowledge of objects is to be gained, Bhartrihari's position has affinities with the Yoga system. Consider, for example, Bhartrihari's view that all reality (objects) exists as a form of consciousness; that reality is essentially undifferentiated (*a-krama*) and that the difference we cognize, the *bheda* in the form of objects, is a form of *avidyā*, ignorance or impure knowledge (3.3.57); and that the knowledge of form is tainted knowledge. So, since one cannot form a correct idea of objects on the basis of perception or verbal knowledge, one should 'examine each object by reasoning, *yuktitaḥ*' (2.141). The Yoga theory of knowledge talks of complete knowledge as a product of deep mental reflection (*dhyāna*). Once this complete knowledge is so gained, we are enabled to see that all objects are merely reflexes of forms constituted in one *sattā*. To grasp this, one has to transcend the *vitarka* (non-discrimination) of *nāma* (name/word), *rūpa* (form), and *jñāna* (knowledge).[28] The argument for this view runs as follows in the *Vakyapadiya*: 'Some consider the universal to be merely something which figures as a common characteristic in our mind and the particular (*dravya*) to be that which figures as differentia. This diversity in objects arises only when they are united in some way. Neither identity nor difference, neither existence nor non-existence of objects is possible if they are not linked with one another. The ultimate view is that it is the "one" which has all powers.'[29]



Grammar : The grand ladder to learning language, albeit a slow one

It is this universal one that is manifest in all the diverse objects: 'Just as a face reflected in water is only called so (that is, a face, without any relation to the water), in the same way a word expresses only the universal manifested by the individual' (3.1.29); and 'it is being (*sattā*) which being differentiated according to the object in which it is present, is called the universal (*jāti*). All words are based on that' (3.1.33).

The theory of *śabda-brahman* is a codification of this theory of meaning. As meaning is an individual cognitive construct and a map of the external, undeniably existing, reality *as mediated through individual consciousness*, language becomes a construct: '*jñānaṁ sarvaṁ śabdena bhāsate*, all knowledge is illumined by words' (1.123); and '*śabdasya pariṇāmo'yam ity-āmanāyavido viduḥ*, those who

know, know this world to be the result of language'(1.120). The concept of *pariṇāma* relates Bhartrihari's thinking to the Buddhists via the Yoga concept of *vikalpa*, while his concept of language as the creator of reality makes him a Vedantin via the principle of Brahman.

This is no mystical conception; it truly expresses both the physical and the linguistic reality. The same word 'apple' designates all the millions of empirically diverse realities (apples); a universal 'appleness' permits this. And in an ascending hierarchy, all universals merge in the one abstract *śabda*, the analogue of the physical *sattā,* the one undifferentiated Reality. This Reality is beyond all appearances, and yet is immanent in all appearances, though limited by particular appearances—'One that is not in one', as John Donne put it in 'The Storm at Sea'. This reality, says Bhartrihari, 'does not exist and it does; it is one and it is many; it is connected and it is separate; it is transformed and it is not' (3.2.13). In an affirmation of the Sankhya-Advaita position, it is asserted that this reality endures, is *anādi* (beginningless), *ananta* (endless), and *a-krama* (non-sequential): 'Just as, when forms (*vikāra*) disappear, it is the gold which remains as the truth (*satya*) of the earrings, in the same way, when transformations (*vikāra*, the physical elements, for instance) disappear, the primordial substance (*prakṛti*) remains' (3.2.15). 'And this *prakṛti* is the expressed meaning of words.'

Subramania Iyer comments: 'All words express Brahman (the universal) differentiated on the basis of limiting factors (*dravya*, substance). Even words like *ātmā*, *brahman*, *tattva* express that *prakṛti* through some limiting factor or other', because, 'that which is beyond all limiting adjuncts (*nirupādhi*) is also beyond the range of words'.[30]

The assertion that *pratibimba* is the basis of the use of words, and the vital *nirupādhi* concept show the deep influence of Buddhist thought on Bhartrihari. Such neat categorization, however, is defied by the Indian tradition, for in this philosophy of language, the *brahmāṇḍa* (universe) is seen as a pulsating continuum of matter in which forms arise and into which they collapse again.

चत्वारि वाक्परिमिता पदानि
तानि विदुर्ब्राह्मणा ये मनीषिणः ।
गुहा त्रीणि निहिता नेंगयन्ति
तुरीयं वाचो मनुष्या वदंति ॥

Speech is known by the wise knowers of the Vedas to be made up of four parts. Three of these—[*parā*, the Shabda-brahman; *paśyanti*, unformed language; and *madhyamā*, mental language]—lie unmanifested in the depths of one's being. It is only the fourth that people speak. —Rig Veda, 1.164.45

Notes and References

23. *Vakyapadiya*, 2.140.
24. Ernst Cassirer, *The Philosophy of Symbolic Forms* (New Haven: Yale, 1955), 99.
25. *Vakyapadiya*, 2.145.
26. Experience filters down through the framework of the *pañca-kośa*s (five bodily sheaths). It reaches the *manas* (mind) and *buddhi* (intellect) through the *jñānendriya*s (senses of knowledge), and the reaction/response is mediated by the *karmendriya*s (senses of action).
27. *Vakyapadiya*, 3.14.325.
28. *Yoga Sutra*, 3.14–20, 3.49–54, 4.18–20.
29. *Vakyapadiya*, 3.1.19–22.
30. *The Vākyapadīya of Bhartṛhari*, chapter 3, part 1, 74.

Shabda-brahman: music of the spheres

CHANDRA

The Cosmos in Western and Indian Thought

Swami Durgananda

(*Continued from the previous issue*)

The hallmark of Indian philosophy is the fundamentality of the uncreated, unbounded, unchangeable, unmovable, invariant, inexpressible, luminous, omniscient, omnipotent pure consciousness—of the nature of the self (*ātmarūpa*). This is common to all orthodox systems of Indian philosophy, despite differences in their modes of expression. According to the Indian systems, 'reality' *precedes* everything, including us, and one can uncover it by un-becoming, by undoing the adventitious, the incidental, the accidental, the unreal, leaving reality in its pristine purity, which roughly translates into moulding one's own character, attitude, or will,[17] and working against egocentric and instinctual forces towards one's infinite source. The 'I' in the Indian system is a secondary product (the fundamental being Reality itself); tertiary are the mind and intellect, which by their nature perceive the real through time, space, and causation, collectively called maya. Mind thus is a limiting adjunct to the infinite knowledge already in the perceiver. But in Western thought, the position of the high-and-mighty judge, the mind, has remained largely unassailed. In it, the correct *description*—a later product constructed by mind and intellect—of the already existing universe is the so called 'reality'. But the *description* of the solar system given by Einstein is based on a very different understanding of reality than that given by Kepler. So it is no wonder that, as the Western system built its large philosophic structure on the shifting sands of the mind and the intellect, its final conclusions appear to be shifting in an ever extending and unending sequence.

Indian Mythology

Having understood that (i) the mind, intellect, and their attendant accoutrements such as logic and reason are impediments rather than aids to the discovery of reality, and that (ii) the reality *precedes* our thinking, waiting to be revealed to us—a revelation held up only by the turning away on *our* part—we can now appreciate the importance of myth, which in most cases is *āpta-vākya*, the testimony of those who have directly perceived the truth.[18]

Myth is a profound answer to profound questions.[19] Myth bypasses the labyrinth of the logical and rational structure of the superficial, conscious mind, the guard of our deeper being, and penetrates inside. It is our inner being that is always open to and needs suggestion; it is in control of, but normally not accessible to, the outer being.[20] Myth is a 'master mechanic', for it knows its business very well, and will effect its intended transformation inside us without our conscious knowledge.

In the West, myth has predominantly performed such useful functions as development of valour, fearlessness, and other virtues; but in India, it has been principally and very effectively used as a vehicle to convey higher, subtler truths that are beyond the grasp of the ordinary, ratiocinating mind. The epics Ramayana and Mahabharata, and the eighteen Puranas have been serving this purpose wonderfully.[21]

Puranic Description of the World

The Bhagavata Purana describes creation as non-separate from the Divine—a quintessentially Hindu view:

> This Maya is His (Vasudeva's), by the infatuation of which men are entrenched in their ego-sense and speak of 'I' and 'mine' … *Dravya* (matter, with the elements of which combinations are made),

> *karma* (impressions of past action, impelling work), *kāla* (time, the substrate of all experience), *svabhāva* (nature, the cause of evolution), and *jīva* (the living soul)—none of these have any existence of their own independent of the Lord.[22]

Time, in Indian thought, has a cyclic nature; whereas it is generally viewed as linear in the West. The Puranas describe cosmic time as follows: one human year is equal to one day of the gods, or celestial day; 360 celestial days make up one celestial year. 12,000 celestial years, consisting of four *yuga*s, make up one *mahāyuga*; and 1,000 *mahāyuga*s are equal to one day of Brahma. A day of Brahma is called a *kalpa*, at the end of which the entire universe is dissolved in *naimittika-pralaya* (occasional dissolution).[23] The next 1,000 *mahāyuga*s comprise a night of Brahma. When Brahma wakes up, the samskaras of the previous *kalpa* stored in latent form become the seed for the next day of Brahma. At the end of 100 years of Brahma, Prakriti dissolves into the primal cause; this is called *prākṛta-pralaya*. Then a new creation arises through a fresh *brahmāṇḍa* (Cosmic Egg), and a new Brahma presides.[24]

This cosmic description of time reflects the alternating periods of activity and quiescence of human life. We may note that there are 'three quiescences' in human life, namely, sleep, death, and samadhi, during which the experience of the world vanishes, and after which the world reappears based on latent impressions.

The Hindu View of the Universe

In the Hindu view, it is memory (latent impressions, or samskaras) that continues across the quiescence—not exactly the 'person', which has a dubious status. This is because the 'person' is sometimes conceived as the aggregate of body and mind, and sometimes as distinct from the body; but both body and mind are constantly changing. To appreciate this, we may reflect upon the amount of change death brings in a 'person'. In the seed that transmigrates across a quiescence are the impressions from which the universe appears—thus in the Hindu view, the universe is a projection, rather than a creation, based on the impressions illumined by existence (*sat*), consciousness (*cit*), and inherent bliss (*ānanda*). Thus after every quiescence, the universe is re-projected. In this view, the underlying consciousness is recognized as fundamental and all-powerful, making all else a mere appearance.

What then is the status of the universe that we see? What of this abundant and vast experience? What of this grand and awe-inspiring symphony?

The answer is, though Brahman is the only reality, the external world is *also* consciousness, a reflection of one's own consciousness: so it appeared to the rishis. The idea that the universe is one living being is common in the Hindu scriptures. The *Purusha Sukta* sings: '*Sahasraśīrṣā puruṣaḥ, sahasrākṣaḥ sahasrapāṭ … puruṣa evedaṁ sarvam, yadbhūtaṁ yacca bhavyam*; This Purusha has a thousand heads, a thousand eyes, a thousand feet … Purusha is all this, what has been and whatever will be.'[25] The scriptures also mention the Virat Purusha (the cosmic person) with trees as his hair, the wind as his breath, the sun as his sight, etc.[26] The *Narayana Sukta* proclaims: '*Viśvaṁ nārāyaṇaṁ devamakṣaraṁ paramaṁ padam … yacca kiñcijjagatsarvaṁ dṛśyate śrūyate'pi vā*; This universe is truly Narayana, the self-effulgent Divinity, the imperishable, the all pervasive … all this, whatever is heard and seen.'[27] And the Bhagavata narrates: 'That Cosmic Being, brilliant like gold, and containing all created beings in potentiality, lay in the Causal Waters for a thousand divine years, enfolded in the Cosmic Shell (*brahmāṇḍa*).[28]

Creation of the Universe

Two important classes of Hindu scriptures, the Vedas and the Tantras, give an illuminating picture of the creation of the universe. The description in the Rig Veda, the most ancient text in the world, is all-encompassing and fundamental (see discussion on the *Nasadiya Sukta* below). The Rig-Vedic hymns are the expressions of the deep realizations of the rishis. In the Upanishads, we find a beautiful imagery of psychophysical sheaths that mediate

our relation to the universe.

The Tantras consist chiefly of practical ways of realization and extend upon the metaphysics of the Vedas. They describe creation from a basic continuum, called *nāda* (primordial vibration), into the observed universe as a sequence of 'splitting' polarities. From *nāda* comes the infinite intentness of that power to create. This is called *bindu* (dynamic point of the potential universe).

> From *bindu* descends … the polarity of *kalā* and *varṇa*. … The term *kalā* [lit., 'partial'] … must mean that aspect of Reality (Śiva) by which it manifests as power (Śakti) for evolving universes and involving them again. … It is the starting point of differentiation. It is here that time, space, thing, attribute etc. are differentiated from an 'alogical' integrated whole (*nāda-bindu*). … in *kalā* … all gradation and gradualness (all ascending and descending series in the cosmic process) have their possibility of appearance. …
>
> This does not mean that we have been already landed in the 'concrete' (*sthūla*) universe. … *Varṇa*, here, does not yet mean 'letter' or 'colour' or even 'class', but only the 'sense' or the 'function form' … of the primordial object projected from 'perfect activity' (*bindu*). …
>
> Then, in the subtle (*sūkṣma*) or vital plane, this polarity manifests *tattva* and *mantra*. [*Tattva* is the essence and *mantra* is the symbol representing it.] … The third and last polarity [is] *bhuvana* and *pada*. *Bhuvana* is the universe as it appears to apprehending and appreciating 'centres', such as we are. … *Pada* (*padyate anena iti*) is the actual formulation (first by mind reaction and then by speech) of that universe.'[29]

'The Sanskrit word *padārtha* is derived from *pada* (the fraction, the divided, the word) and *artha* (meaning); thus matter is the meaning behind the word. It is said that *śabda* (primordial vibration) creates or manifests everything.'

We have reviewed the description of creation according to the Tantras in order to appreciate that the world is a 'movement' in the otherwise pacific, unmovable consciousness. (Even a static body such as a stone is a 'movement' in consciousness, for (i) time is flux in consciousness, and (ii) the impression of the stone on the mind is a *vṛtti*, a perturbation in consciousness.)

We must appreciate that the underlying, inexpressible essence is the referent for myth, symbol, and language. It is the nature of mind to differentiate. When we approach close to it, Truth begins to appear in paradoxes or contradictions to the dichotomizing mind—because it sees duality where there is none. This is why Western philosophy is at crossroads. This is also why we come across contradictions in the scriptures.

Against this backdrop, we note how Sri Ramakrishna steered clear of controversy. He said: 'He who has seen God knows really and truly that God has form and that He is formless as well. He has many other aspects that cannot be described.'[30]

He would often sing: 'Who is there that can really understand what Mother Kāli is? Even the six darśanas are powerless to reveal Her' (106).

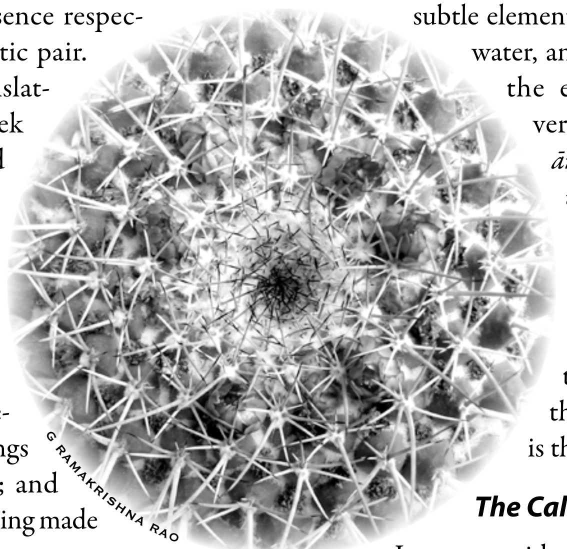
We also read the words of Swami Vivekananda: 'Again, the last word [of the Vedas] gave us one universe, which through the senses we see as matter, through the intellect as souls, and through the spirit as God.'[31]

Again: 'The universe is thought, and the Vedas are the words of this thought. We can create and uncreate this whole universe. Repeating the words, the unseen thought is aroused, and as a result a seen effect is produced' (7.47).

Logos, which we have likened to the divine yajna, is a Greek term meaning 'word', 'ratio', or 'reason'. How do these three meanings relate? Logos is the universe, the external expression, just as a word is an expression of the idea behind it. 'Ratio' means division, which is implicit in any expression. We

have just seen how the Tantras have made plentifully transparent the necessity of splitting (division) before sprouting of an expression. 'Reason' is causality, as it appears to the human mind within creation. The English word *ratiocinate* (to reason logically) in its Latin form means 'to deliberate, to calculate', and is derived from the root *ratio*, 'to differentiate'.

It must be noted that Logos (the ordered, the apparent, the expressed) and mythos (the unordered, the underlying, the experienced) stand for the construct and the essence respectively, and form a linguistic pair. When the Bible was translated into English, the Greek word *Logos* was translated as *Word*. Thus the beginning of the Gospel according to John reads: 'In the beginning was the Word, and the Word was with God, and the Word was God. He was in the beginning with God; all things were made through him; and without him was not anything made that was made.'[32]



Word here obviously represents *śabda* or *nāda* as described above—the essence from which the world has evolved. Logos must refer to the cosmos. The term *cosmos* is derived from the Greek *kosmos*, order, and means the universe seen as a well-ordered whole. (The word *universe* is derived from the Latin *universum*, meaning combined into one, whole—from *uni*, one, and *versus*, turned.) Because of the West's excessive obsession with the apparent, and its unwillingness to turn to the real, we find the Word and the Logos perpetually mixed in the Western tradition.

The Gospel according to John continues: 'In him was life, and the life was the light of men. The light shines in the darkness, and the darkness has not overcome it' (1.4–5).

This is clearly a distant echo of the *Nasadiya Sukta*, venerated for thousands of years in India:

nāsadāsīnno sadāsīt tadānīṁ nāsīdrajo
no vyomā paro yat,
kimāvarīvaḥ kuha kasya śarmannambhaḥ
kimāsīdgahanaṁ gabhīram

Existence was not then, nor non-existence,
The world was not, the sky beyond was neither.
What covered the mist? Of whom was that?
What was in the depths of darkness thick?[33]

The *Taittiriya Upanishad* describes the universe as emanating from Brahman as a succession of subtle elements: space (*ākāśa*), air, fire, water, and earth. It also describes the experience of the universe through five sheaths: *ānandamaya* (the blissful), *vijñānamaya* (the intellectual), *manomaya* (the mental), *prāṇamaya* (the vital), and *annamaya* (the gross). The world is thus a self-revelation of the indwelling Deity, and is thus divine.

The Call of the Cosmos

Let us consider the following four phenomena: conscious death, willed death, willed birth, and conscious birth. Conscious death, which means remaining conscious *during* dying, can be achieved by an ordinary sadhaka who does his or her sadhana sincerely. Willed death, which means dying whenever one wants (barring, of course, suicide), can be achieved only by saints. Willed birth, which means taking birth upon one's own wish as opposed to being born haplessly as a consequence of one's karma, is taken by avataras. Conscious birth, meaning remaining conscious *during* birth, is very rare, and occurs only in the case of an *avatāra-variṣṭha* such as Sri Ramakrishna. It is said that Sri Ramakrishna was conscious during his birth—that immediately on leaving the womb, he had rolled into the fireplace and become covered with ashes, the mark of Shiva.[34] And Indian phi-

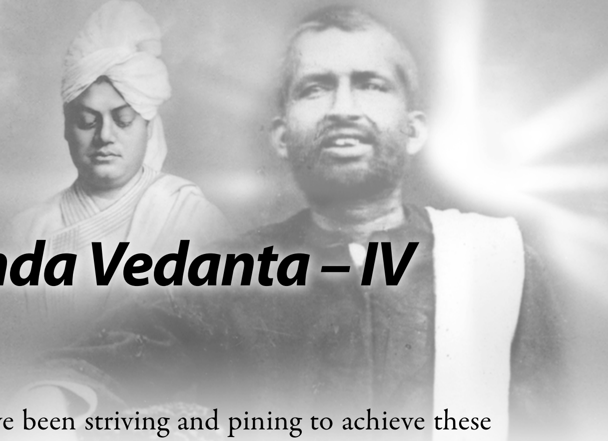
losophy looks to the testimony of such beings to verify its claims.

The world which, in the wake of Sri Shankaracharya, was described for nearly a thousand years by Indian thinkers as a figment of the mind to be escaped from and to be despised, the world which appears as purposeless matter to the West, has been described by Sri Ramakrishna as lila, divine play. Sri Ramakrishna saw the world as God's will. He saw in the world a sport, a creative expression spontaneously arising from God himself, and he saw God in everything. Such is the glory of the universe.

The world is pure knowledge personified, it is an expression of God's unsullied love, and it is also a sacrifice of the primal Person. These three conceptions reveal three routes through which, with the world itself as an instrument, we may reach our final, or rather, our original state of divinity. The three routes are discrimination (reason in action), devotion to God (love in action), and sacrificing our limited personality (detachment in action)—each of these three may be thought of as the principal mode of sadhana in jnana yoga, bhakti yoga, and karma yoga respectively, the other two being attendant modes. The world is a song in God's mind. It is a poem in the seer's eyes. It is Brahman at play—thus say the wise.

Notes and References

17. These may be understood to correspond to karma yoga, jnana yoga, and bhakti yoga respectively.
18. Reason or rationality does perform valuable functions: (i) it is of great utility in the *vyāvahārika* (the apparent) plane for production of food and other means of living (*abhyudaya*), and (ii) in the *pāramārthika* (the spiritual) plane, it is used to generate *viveka* (discrimination) and *vairāgya* (dispassion). With the appearance, development, and maturity of the latter two, mind will vanish—in the same manner as a fire burns itself out—and reality will surface.
19. Barbara C Sproul, *Primal Myths* (New York: Harper Collins, 1991), 1.
20. On the subject of suggestion, see Joseph Murphy, *The Power of Your Subconscious Mind* (Eaglewood Cliffs: Bantam, 1985).
21. For an instructive discussion on the types of religious languages, see Swami Bhajanananda, 'Sri Ramakrishna and the Vedic Ideal', *Prabuddha Bharata*, 85/3 (March 1980), 83–90.
22. Bhagavata, 2.5.13–14.
23. By these calculations, a *kalpa* would last 4,320,000,000 earth years.
24. Swami Mukhyananda, 'The Unique Space-Time and Historical Sense of the Hindus', *Prabuddha Bharata*, 97/6 (June 1992), 253.
25. *Taittiriya Aranyaka*, 3.3.12.
26. Bhagavata, 2.10.15–30; also, *Aitareya Upanishad* 1.1.4.
27. *Taittiriya Aranyaka*, 4.10.13.
28. Bhagavata, 3.6.6.
29. Swami Pratyagatmananda, 'Philosophy of the Tantras', in *The Philosophies*, ed. Haridas Bhattacharyya, vol 3 of *The Cultural Heritage of India* (Calcutta: Ramakrishna Mission Institute of Culture, 1953), 442–4.
30. M, *The Gospel of Sri Ramakrishna*, trans. Swami Nikhilananda (Chennai: Ramakrishna Math, 2002), 191.
31. *The Complete Works of Swami Vivekananda*, 9 vols (Calcutta: Advaita Ashrama, 1–8, 1989; 9, 1997), 2.252–3.
32. John, 1.1–3.
33. Rig Veda, 10.129.1; trans. Swami Vivekananda, *Complete Works*, 6.178.
34. Swami Saradananda, *Sri Ramakrishna the Great Master*, trans. Swami Jagadananda (Madras: Ramakrishna Math, 1983), 46.



The Many-splendoured Ramakrishna-Vivekananda Vedanta – IV

Dr M Sivaramkrishna

There is a Mahatma Gandhi wave today. Not just books, but movies too are evoking, in their own way, his perennial relevance. Oxford has published—to name only two—*Post-modern Gandhi and Other Essays* by Llyod I Rudolph and Susanne Hoeber Rudolf (2006) and *Debating Gandhi: A Reader* edited by A Raghuramaraju (2006). Cambridge University Press has brought out the South Asian Edition of *Gandhi as Disciple and Mentor* by Thomas Weber (2007).

Naturally, I was curious to know whether there are references to Sri Ramakrishna and Swami Vivekananda in such volumes. Though I could not find any in the above, to my luck, I found quite a few books where the Great Master and his illustrious disciple figure. First, *Gandhi and Gutierrez: Two Paradigms of Liberative Transformation* by John Chathanatt, professor at Vidyajyoti College of Theology, Delhi (New Delhi: Decent, 2004). The author did his doctoral work on 'ethics and society' at the University of Chicago and has degrees in mathematics, philosophy, economics, and theology.

This is a study of what has come to be called 'liberation theology'. Gutierrez 'attempted to reflect and articulate a theology of liberation in the context of the exploitation prevalent in Latin America, and has been rightly called "the Father of Liberation Theology"' (from the blurb). Chathanatt compares Gandhiji with Sri Ramakrishna and says: 'Gandhi's pursuit of Truth was a total commitment. The commitment involved every moment of his being and existence. Like Ramakrishna, the mystic, he was *drunk* with the religious experience of the reality of Truth' (83). And he quotes from Gandhiji's *Autobiography*: 'What I want to achieve,—what I have been striving and pining to achieve these thirty years,—is self-realization, to see *God face to face* [emphasis added], to attain *Moksha*. I live and move and have my being in pursuit of this goal. All that I do by way of speaking and writing, and all my ventures in the political field, are directed to this same end.'

The striking coincidence is in the very first sentence: seeing God 'face to face'. We recall that the Mahatma, in his short and suggestive foreword to the *Life of Sri Ramakrishna* (Calcutta: Advaita Ashrama), says that the Great Master's life 'enables us to see God face to face'. In fact, this foreword is a model of brevity which, properly enlarged and expanded, triggers the most amazing revelations regarding the unique significance of the Great Master. The foreword was written on 12 November 1924 at Sabarmati.

The year 1924 was quite significant in Gandhiji's life. He was in prison; on 4 February came the order for his release. On 17 September commenced his twenty-two day fast for Hindu-Muslim unity. And in November he wrote the foreword. I do not have, yet, further details about how the foreword by the Father of the Nation came to be written. (Perhaps they are lying somewhere in his vast published corpus.)

The key point that Gandhiji noted reiterates two of Ramakrishna's tenets: (i) '*jivaner uddeshya ishwar-labh*; the purpose of life is God-realization'; and (ii) '*karma upay, uddeshya nay*; karma is the means, not an end'. These two are planks of Gandhiji's own life, in whatever dimension it manifested—the most significant being the political arena. In fact, liberation theology also stresses the spiritual dimension as the driving force for effective trans-

formation of societies from acquisitive to sharing models (to simplify a complex matrix).

Before I cite other aspects of these two figures, I should first mention another book which has an intriguing title: *Inhaling the Mahatma* by Christopher Kremmer, who was educated at the University of Canberra and is a journalist by profession (Delhi: Harper Collins, 2007). This is a fascinating 'travelogue'. Embarking on a yatra, says Kremmer, 'I have chosen to tell India's story in the context of my own changing life there.' He 'discerned' that there is 'the fundamental difference between the tolerant Hinduism of the majority and the less appealing kind practiced by power-hungry politicians'. And, making a 'confession', he says, 'I have been personally moved and inspired by the Hindu faith and philosophy, and fully expect it increasingly to influence the world in the twenty-first century' (x).

No wonder that he is an ardent admirer of the Great Master. He goes on a visit to Varanasi, and, in that context, says: 'For my trip there I'd taken along several books on one of India's most revered saints, Sri Ramakrishna, including Christopher Isherwood's *Ramakrishna and His Disciples*.' Describing him as a bhakta, Kremmer says: 'Throughout his life he would be blessed or afflicted [*sic*] with this singular capacity for experiencing overwhelming joy' and was 'recognized from an early age as a spiritual phenomenon' (335–6).

Kremmer quotes Romain Rolland on Sri Ramakrishna's experience of the truth of other religions and says: 'In his quest for enlightenment, Ramakrishna hurled himself against barriers and taboos of all kinds' (336). Ramakrishna 'plucked three beautiful fruits from the tree of Knowledge—Compassion, Devotion, and Renunciation'; and eventually, he 'came to see God in everything and everyone, from the most pious holy man to the hypocrite and the criminal' (337). When Kremmer sees Varanasi, he is reminded of Ramakrishna's visit there and his vision of Shiva himself giving moksha to all those who died in that holy *kshetra*. Even 'a century after Ramakrishna visited Varanasi it was still easy to understand his excited embrace of the city,' says Kremmer.

He draws attention to Swamiji also, succinctly summarizing his contribution: 'Indians educated in Britain during the Raj were influenced by modern ideas of democracy and secular government and, together with Indian philosophers such as Swami Vivekananda, Swami Dayananda Saraswati and Sri Aurobindo, participated in a broad Hindu reform movement that became known as the Bengali Renaissance. *A passionate interest in new ideas and a solid faith in ancient ones formed the two banks between which the river of Indian nationalism flowed*' (78; emphasis added). Discussing issues of inter-religious faith, he says: 'Swami Vivekananda, one of Hinduism's great missionaries, once famously said that Hindus accept all religions as true' (191).

So far, we have seen that the Great Master and his illustrious disciple figure in contexts of social/religious movements and the archetypal motifs embedded in the City of Light, Kashi. Moreover, interfaith communion and a sense of oneness also surface. Both are regarded as bridges of nationalist upsurge, linking the old and the new. Both the secular and the spiritual realms felt the impact of these two towering figures. One overall result is service embodied in liberation theology.

Yet, the secular is never free from destructive elements. Gandhiji felt the fury of this in full force on the eve of India's independence and especially in the communal violence in, more than any other place, Noakhali. One can, in theological terms, call this the *karala nritya*, the cosmic destructive dance, of Mother Kali. Bringing out the impact of Noakhali on Gandhiji, V Ramamurthy imaginatively puts it thus:

> The sprawling old city of Calcutta on the river Hooghly, *a child of the Ganga, the famous Belur Math of Sri Ramakrishna*, the spiritual Province of Bengal, hallowed land of Kabir [*sic*] and of Vishwabharati, that dream university fostered by the great visionary soul and noble poet of India, Gurudev Rabindranath Tagore—what fragrant store of precious memories these and their related associations evoked for Gandhiji, memories set in the

honey-comb of his mind like precious sweet stock lovingly gathered and carefully cherished over several decades of the past! Now, all of a sudden, incomprehensibly terrible forces released from one knew not where, were threatening .[1]

It is absolutely apt that in the memories that animated Gandhiji, Belur Math should figure. Gandhiji, as we noted above, had admiration for Sri Ramakrishna transcending the bounds of articulation. No wonder that, as the same author in another context notes, Gandhiji felt so much love for Bengal that with the help of Nirmal Kumar Bose, he started 'learning the language of Kabir [Kubir?], Sri Ramakrishna Paramahamsa, the Deshabandhu, and of course Gurudev' (135).

Rajmohan Gandhi, in his 745-page study of Gandhiji, makes two references to Swamiji. The first is concerned with the period 1901–6. During this period, says the author, Gandhiji visited Calcutta and 'met many of Bengal's intellectual and political leaders and found a love for Bengali music. Eager to call on Swami Vivekananda, he walked "with great enthusiasm" the long path to Belur Math, only to be told that the Swami was "in his Calcutta House, being ill, and could not be seen". … Yet this disappointment was nothing compared with Gandhi's horror at the "rivers of blood" beside Calcutta's Kali temple.'[2]

Rajmohan Gandhi also draws attention to the fact that in Yeravada prison, 'while giving six hours a day to books, he went through scores,' and among them figured 'the writings of Swami Vivekananda' (276).

Interestingly enough, in a recent massive study of Hinduism from the Gandhian perspective, the Great Master figures in six or seven contexts (and this is teleologically quite appropriate). The author, Dr M V Nadkarni, former professor at the Institute of Social and Economic Change (ISEC), Bangalore, was later Vice Chancellor of Gulbarga University. His book—*Hinduism: A Gandhian Perspective* (Delhi: Ane, 2006)—runs to 510 pages and is an invaluable study of this most complex religion.

The most fascinating context in which Sri Ramakrishna is placed correlates the Great Master and Gandhiji. Chapter six, entitled 'Dynamics of Hinduism: Modern Phase', traces the contribution of several savants to the Hindu renaissance. In that context, Nadkarni discusses the work of Dayananda Saraswati (1824–83) who 'brought out the positive features of Hinduism and explained his philosophy'. Yet, 'The Arya Samaj has become more aggressive than the Brahmos and the Prarthana Samaj'. Therefore, says Nadkarni, 'lest this should go too far, a corrective was provided by Ramakrishna Paramahamsa and Mahatma Gandhi'. This is an interesting perspective: though Sri Ramakrishna never criticized or corrected but by his own gentle impact fulfilled the path even as it retained its essentials, it is virtually the first time that Gandhiji gets bracketed with Sri Ramakrishna in the context of providing checks and balances. Elaborating this, Nadkarni points to Sri Ramakrishna's experience of the unity of religions:

> The God that Ramakrishna saw was not a Hindu God, a Muslim God or a Christian God. His God was common to the whole Universe. He realized the unity of God and taught it. He also experimented. He wanted to have a first-hand experience of the feeling, of the realization that honestly practised, all religions lead one to the ultimate realization of the same God. He had already practised as a devout Hindu and had realized God.

Nadkarni then comments on the Mission:

> [The] Ramakrishna Mission which Swami Vivekananda founded has, therefore, avoided all

> narrow parochialism and preached Hinduism in its universal essence. The teachings of Ramakrishna and Vivekananda yield no scope for any hatred for 'other' religions, which in fact is true of Hinduism as a whole (292–93).

The author adds that the bhakti movement also featured many comparable elements. But there is one difference. Ramakrishna felt in his very pulse the remarkable impact of the emotions associated with the path of devotion. The emotional motifs find palpable manifestations in the Great Master. This uniqueness has been commented upon by students of the bhakti rubric of Hinduism.[3]

In an interesting volume entitled *Emotions in Asian Thought: A Dialogue in Comparative Philosophy*, June McDaniel comments:

> In the later *bhakti* (devotional) traditions, passion is said to burn the hearts of devotees, causing the devotee to be 'on fire' for the god. Some saints have said that their bodies would burn with fever for the deity for months or years on end.

McDaniel cites the Great Master's example:

> Ramakrishna Paramahamsa, a recent Bengali Shakta saint, could not touch other people during his meditation on the deity, because his body was physically burning from passion, and he had to wear a sheet when approaching others.[4]

Gandhiji too, was a great votary of bhajans, one of the most important elements of bhakti.

Professor Nadkarni's book gives a concise and clear account of Holy Mother Sri Sarada Devi's significance among 'women gurus and saints in the modern phase of Hinduism'. He traces the various phases of Mother's life which culminated in her role as *Sangha Mata*, and how Holy Mother inspires countless women, in both East and West to lead a life modelled after her teachings.

Finally, one more instance regarding the role of emotions appears in the context of the Shaivaite Tantric text *Sri-vijnana-bhairava-tantra*. In his commentary on this text, Swami Satyasangananda Saraswati instances Sri Ramakrishna. The comment is on verse 143, in which Devi asks a question of Shiva: 'Thus (Devi) said, "O great Lord, (tell me) in the established order who would be invoked and what would be the invocation? Who is to be worshipped or meditated upon and who is to be gratified by that worship?"'

The answer is, 'in reality' both the worshipped and worshipper are the same. How this truth is demonstrated by Sri Ramakrishna is pointed out by Swami Satyasangananda:

> In heightened states of trance Paramahamsa Ramakrishna would often wave the lights over himself instead of the Goddess Kali, whom he was worshipping. He would put the garland on himself rather than on the statue of Kali. He would put the prasad of sweets into his own mouth, instead of placing them in front of her. He did not see any difference between himself and the Goddess. *This is precisely what Devi is implying* when she asks: when that state arises then who is the worshipper and who is the worshipped? What is the distinction or the difference between the two?[5]

Isn't it intriguing that what the Devi asks and Shiva answers find precise confirmation from the modern Shiva, Sri Ramakrishna?

Researchers will find the following areas for further enquiry extremely fruitful:

(i) The Great Master and the Mahatma;
(ii) The modern Indian renaissance: Towards new confirmations and configurations;
(iii) Sri Ramakrishna's confirmation of tantric truths; and
(iv) Role of emotions in the spiritual quest.

References

1. *From the Pages of The Hindu: Mahatma Gandhi, The Last 200 Days* (Chennai: Kasturi, 2005), 59; emphasis added.
2. Rajmohan Gandhi, *Mohandas: A True Story of a Man, His People, and an Empire* (Penguin, 2007), 99.
3. See, for example, Edward C Dimock, *The Place of the Hidden Moon* (University of Chigago, 1989); Indian reprint: Motilal Banarsidass.
4. *Emotions in Asian Thought: A Dialogue in Comparative Philosophy*, ed. Roger T Ames, Joel Marks, and Robert C Solomon (Albany: SUNY, 1995; Indian reprint: Satguru, 1997), 45.
5. Swami Satyasangananda Saraswati, *Sri Vijnana Bhairava Tantra: The Ascent* (Munger: Yoga Publications; 2003), 403–4; emphasis added.

Bijaganita *of Bhaskaracharya*

Dr Amartya Kumar Dutta

> What would have been Fermat's astonishment if some missionary, just back from India, had told him that his problem had been successfully tackled there by native mathematicians almost six centuries earlier! — *André Weil*

The mathematician Pierre de Fermat (1601–65) is regarded as the father of modern number theory, and the problem referred to above by André Weil (1906–98)—one of the giants of 20th century mathematics—in his book *Number Theory: An Approach through History from Hammurapi to Legendre*, has a grand history. It was posed by Fermat in 1657 as part of his efforts to kindle the interest of contemporary mathematicians in the abstract science of numbers. The problem was to find all integers (or whole numbers) x, y which satisfy the equation $Dx^2 + 1 = y^2$, where D is a fixed positive integer which is not a perfect square. The unexpected intricacy of the problem can be felt from the case $D = 61$. The smallest solution (in positive integers) of the equation $61x^2 + 1 = y^2$ is $x = 226{,}153{,}980$, $y = 1{,}766{,}319{,}049$. In his challenge, Fermat had specifically highlighted this case ($D = 61$).

The above problem turned out to be of paramount importance in algebra and number theory. It fascinated some of the greatest mathematicians of modern Europe like Euler (1707–83) and Lagrange (1736–1813). Powerful theories and techniques emerged out of the researches centred around this equation.

A thousand years before Fermat, the ancient Indian mathematician-astronomer Brahmagupta (628 CE) had investigated the same problem, and came up with a brilliant composition law, or *bhāvanā*, on the solution space of the more general equation $Dx^2 + m = y^2$ called *vargaprakṛti* (square-natured). Brahmagupta's work anticipates several basic principles of modern number theory and abstract algebra. Using Brahmagupta's rule, subsequent Indian algebraists developed an astonishing algorithm called *cakravāla* which gives a complete solution to the problem. The algorithm was discovered by the 11th century—a work of 1073 CE quotes the algebraist Jayadeva's solution to the problem!

The algebra text *Bijaganita* (1150 CE) of Bhaskaracharya (b. 1114 CE) gives a brief but lucid description, in Sanskrit verses, of the *bhāvanā* law followed by the *cakravāla* algorithm for solving Fermat's equation. The method is illustrated by two difficult examples including the peculiar example $D = 61$. No wonder that glowing tributes were paid by European scholars after a translation of *Bijaganita* was published. The German mathematician Hankel wrote about the *cakravāla* method: 'It is beyond all praise: it is certainly the finest thing achieved in the theory of numbers before Lagrange.'

Bhaskaracharya's verses on the topic *vargaprakṛti* occur in chapter six of the book. The preceding fifth chapter discusses integer solutions to the linear equation $ax - by = c$, a problem with applications in astronomy and calendar-making. The problem had been solved by Aryabhata (499 CE) by a method called *kuṭṭaka* (pulverization), which involves a subtle idea resembling Fermat's celebrated 'principle of descent'.

There are various other interesting examples of problems involving integer solutions in Bhaskaracharya's *Bijaganita*. From the verses, one can get a glimpse of the thrill and delight that the ancient Indian algebraists felt in handling such difficult number-theoretic problems. Even today, not many high-school students (or even college students) in India are familiar with this important branch of mathematics.

Bijaganita also covers topics in basic algebra that are now familiar to high-school students: negative

numbers and zero, variables (unknowns), surds and the fundamental operations with them, solutions of simultaneous equations in several unknowns, and the solution of the quadratic equation by the method of 'elimination of the middle term' (or 'completing the square')—an idea with far-reaching consequences in mathematics. As in modern school texts, interesting concrete examples are given to illustrate applications of the principles.

Bhaskaracharya took the bold step of introducing infinity in mathematics and defining rules of interactions with usual numbers: $\infty + x = \infty$ and $\infty - x = \infty$. The idea of adjunction of infinity has now been put on a firm footing in several branches of higher mathematics like analysis or the valuation theory in commutative algebra and number theory.

The verses in *Bijaganita* are in the *anuṣṭup* metre. Ancient Indians had the perception that metrical form has greater durability, power, intensity, and force than the unmetrical, and invariably recorded all important knowledge in verse form. It could be exciting for a modern reader to watch how Bhaskaracharya moulds the Sanskrit language to present technical terms and hard results of mathematics in poetic verse!

Touches of mythological allegories enhance the charm of Bhaskaracharya's *Bijaganita*. While discussing properties of the mathematical infinity, Bhaskaracharya draws a parallel with Lord Vishnu, who is referred to as Ananta (endless, boundless, eternal, infinite) and Achyuta (firm, solid, imperishable, permanent): 'During *pralaya* (cosmic dissolution), beings merge in the Lord and during *sṛṣṭi* (Creation), beings emerge out of him; but the Lord himself—the Ananta, the Achyuta—remains unaffected. Likewise, nothing happens to the number infinity when any (other) number enters (i.e., is added to) or leaves (i.e., is subtracted from) the infinity; it remains unchanged.'

The use of a mystic metaphor to explain the mathematical principle $\infty \pm x = \infty$ reflects the vibrant culture of a bygone era. Perhaps its spiritual culture had prepared the Indian mind for, and probably suggested to it, the concept of the mathematical infinity (or the zero!) with its curious properties.

Again, in order to emphasize the importance, power, and profundity of algebra, Bhaskaracharya begins the treatise with an invocation involving an interesting pun on the words *sāṅkhyāḥ* (the Sankhya philosophers as well as the experts in *saṅkhyā*, the science of numbers), *bīja* (root/cause as well as algebra) and *vyakta* (the manifested universe as well as the revelation of an unknown quantity). Thus, through the opening verse, Bhaskaracharya venerates the Unmanifested—the self-existent Being of the Sankhya philosophy—who is the originator of intelligence and the primal Cause of the known or manifested universe; and, through the very same words, he pays tribute to the wise mathematician who, using algebra, solves a problem (i.e., reveals or manifests an unknown quantity)!

The importance of algebra is reiterated at the end of *Bijaganita*. Bhaskaracharya remarks that algebra is the essence of all mathematics, is full of virtues and free from defects, and that cultivation of algebra will sharpen the intellects of children. He concludes with the exhortation '*paṭha*, *paṭha*' (learn it, learn it) for the development of intelligence.

In this connection, I may mention here that one of our greatest contemporary mathematicians, Shreeram S Abhyankar (b. 1930), acknowledges the influence of Bhaskaracharya during his formative years. Abhyankar fondly recalls how his father (S K Abhyankar) used to teach him mathematics by reciting to him lines from Bhaskaracharya's text *Lilavati* and how he used to memorize them when he was around ten years of age.

Bijaganita not only makes us aware of the great advancements made by ancient Indian algebraists, it also gives us a feel for the charming atmosphere in which mathematical research and discourse—at both basic and advanced levels—used to take place in ancient times. While a study of *Bijaganita* will be enriching and inspiring for all cultured students of mathematics, a careful analysis of the treatise will also provide valuable insights to historians and scholars in general.

Grisha Perelman: The Madness of Knowledge

Br. Brahmachaitanya

GRISHA PERELMAN has won the Fields Medal for proving the Poincaré conjecture to be true. This fact is known to virtually every mathematician today. But does this have any relevance outside mathematics? This article aims at justifying an affirmative answer.

Bertrand Russell described mathematics as having a beauty 'cold and austere that appeals to none of our weaker senses'. The truth behind this statement is perhaps what has led to the popular image of a mathematician as one who is absent-minded, lost in his own world. The beauty and sublimity of the subject, like the beauty and sublimity in classical music and art, is something that is usually inaccessible to the untrained mind. Why then, should one choose *Prabuddha Bharata*—clearly not a forum for such esoteric pursuits—as a platform for describing the achievements of the elusive genius that Perelman most certainly is? The answer lies in the word 'elusive'. Perhaps 'reclusive' is a better word to describe Perelman, for, like all recluses, he is averse to public attention.

Who then is Grisha Perelman? An essayist in Wikipedia observes:

> In 2006, Perelman was awarded the Fields Medal for his contributions to geometry and his revolutionary insights into the analytical and geometric structure of the Ricci Flow. The Fields Medal is widely considered to be the top honor a mathematician can receive. However, Perelman declined to accept the award or appear at the congress [International Congress of Mathematicians].
>
> On December 22, 2006, the journal *Science* recognized Perelman's proof of the Poincaré Conjecture as the scientific 'Breakthrough of the Year', the first such recognition in the area of mathematics.

Here we are face to face with an enigmatic personality. How is it that a person of this stature refused the highest honour that the mathematical world has to offer? Political equations have nothing to do with this decision. What then is the reason for the refusal? The answer is that Perelman is simply disinterested in public life. As he told a reporter: 'I do not believe anything I say can be of the slightest public interest.' *The First Post* commented that this statement reveals Perelman to be a most elusive person, a genuine celebrity with no interest in celebrity life.

The Person

A brief biography will help us appreciate what we are up against.[1] Grigori Yakovlevich Perelman was born on 13 June 1966 in St Petersburg (then Leningrad), Russia. He did not plan to become a mathematician. Perelman's father, who was an electrical engineer, encouraged his interest in maths. By the time he was fourteen, he was the star performer of a local maths club. In 1982, Perelman earned a perfect score and the gold medal at the International Mathematical Olympiad in Budapest. He also had a passion for opera. His mother, a maths teacher at a technical college, played the violin and began taking him to the opera when he was six. By the time Perelman was fifteen, he was spending his pocket money on records. At Leningrad University, which Perelman entered in 1982 at the age of sixteen, he took advanced classes in geometry and solved an important problem posed by Yuri Burago, a mathematician at the Steklov Institute, who later became his PhD adviser. 'There are a lot of students of high ability who speak before thinking,' Burago said. 'Grisha was different. He thought deeply. His

answers were always correct. He always checked very, very carefully.' Burago added, 'He was not fast. Speed means nothing. Math doesn't depend on speed. It is about *deep*.'

This is perhaps the right juncture for a parenthetical comment about depth. The concept eludes precise definition, and like other profound concepts, is subjective in its essence. The depth of an idea is usually gauged by judging how fundamentally it affects the subject. A new idea is regarded as deep if it ties together a lot of mathematics, that is, if it unifies and provides a fundamental insight that organizes a large body of knowledge.

At the Steklov in the early nineties, Perelman became an expert on the geometry of Riemannian and Alexandrov spaces—extensions of traditional Euclidean geometry—and began to publish articles in leading Russian and American mathematics journals. In 1992, he was invited to spend a semester each at New York University (NYU) and Stony Brook University. He was pleased to be in the United States, the hub of the international mathematics community. He wore the same brown corduroy jacket every day and told friends at NYU that he lived on a diet of bread, cheese, and milk. Some of his colleagues were taken aback by his fingernails, which were several inches long. 'If they grow, why wouldn't I let them grow?' he would say when someone asked why he didn't cut them.

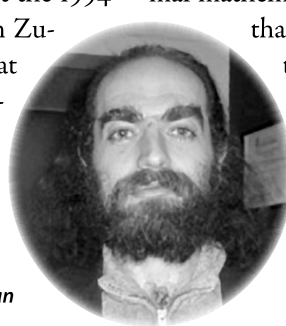
In 1993, he began a two-year fellowship at Berkeley. While he was there, Richard Hamilton from Cornell University gave several talks on campus, and in one he mentioned that he was working on the Poincaré conjecture using a technique called Ricci flow. By the end of his first year at Berkeley, Perelman had written several strikingly original papers. He was asked to give a lecture at the 1994 International Mathematical congress, in Zurich, and was invited to apply for jobs at Stanford, Princeton, and the University of Tel Aviv. At Berkeley he found himself returning again and again to Hamilton's Ricci-flow equation and the problem that Hamilton thought he could solve with it. Some of Perelman's friends noticed that he was becoming more and more ascetic. Visitors from St Petersburg who stayed in his apartment were struck by how sparsely furnished it was. Others worried that he seemed to want to reduce life to a set of rigid axioms. When a member of a hiring committee at Stanford asked him to include his curriculum vitae with requests for letters of recommendation, Perelman baulked at the idea: 'If they know my work, they don't need my CV,' he said. 'If they need my CV, they don't know my work.' Ultimately, he received several good job offers. But he declined them all, and in the summer of 1995 returned to St Petersburg, to his old job at the Steklov Institute, where he was paid less than a hundred dollars a month. At twenty-nine, Perelman was firmly established as a mathematician and yet largely unburdened by professional responsibilities. He was free to pursue whatever problems he wanted to, and he knew that his work, should he choose to publish it, would be shown serious consideration. The Internet made it possible for Perelman to work alone while continuing to tap a common pool of knowledge.

The Poincaré Conjecture

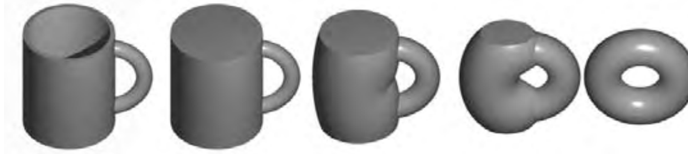
A few words regarding the problem that Perelman was attempting are in order. The conjecture asserts that if any loop in a locally three-dimensional space can be shrunk to a point without ripping or tearing either the loop or the space, this space is equivalent to a sphere in four dimensional space, given by the equation $x^2 + y^2 + z^2 + w^2 = 1$.

The conjecture was first made by the French mathematician Poincaré (often regarded as the last great universalist in mathematics) in 1904. In formal mathematical language, the conjecture states that every simply connected (a technical term explained below) closed 3-manifold is homeomorphic to a 3-sphere.

The conjecture is fundamental to topology, a branch of mathematics that deals with shapes, sometimes described as rubber-sheet geome-



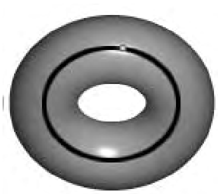
Grigori Perelman



A mug and a tube are topologically equivalent, as each can be moulded into the other without tearing it apart

try—meaning that we are allowed to deform the object of study into any shape so long as we do not tear it. To a topologist, a sphere, a cigar, and an oddly shaped piece of dough are all the same because they can be deformed into one another (assuming they are made of some mouldable material like clay). Likewise, a coffee mug and a bicycle-tube are also the same because each has one 'hole', but they are not equivalent to a sphere. Let us think of a loop sitting somewhere on a sphere. If such a loop starts to shrink, it will always end up as single point. In contrast, if it is draped round the ring of a bicycle-tube, it cannot be shrunk to a point without leaving the bicycle-tube. A space where loops can be contracted to a point are called 'simply-connected'.

Poincaré suspected that this loop test would always reveal if a given surface is spherical. But while it obviously worked with the locally two-dimensional surfaces of balls in our world, he couldn't prove it would for locally three-dimensional surfaces, known technically as 3-manifolds. Two-dimensional manifolds were well understood by the mid-nineteenth century. But it remained unclear whether what was true for two dimensions was also true for three. Poincaré proposed that all closed (i.e. of finite extent), simply connected, three-dimensional manifolds—those which lack holes like the bicycle tube—were spheres. Thus, the conjecture is about the nature of space. This conjecture was generalized to higher dimensions, and surprisingly, solutions were provided by Steve Smale for dimensions greater than 4 in 1961, and then by Mike Freedman for dimension 4 in 1982. Both won Fields Medals. But Poincaré's original conjecture (for dimension 3) remained open.



A loop on a sphere can be shrunk to a point without leaving the surface (below); not so a loop on a tube (left)

The Fields Medal: Awarded and Refused

In 2000, the Clay Mathematics Institute, a private foundation that promotes mathematical research, named the Poincaré conjecture as one of the seven millennium problems (or most important outstanding problems) in mathematics and offered a million dollars to anyone who could prove it. In 2002–3, Perelman announced that he had solved the Poincaré conjecture. After posting a few short papers on the Internet and making a whirlwind lecture tour of the United States, Grisha Perelman withdrew to Russia in the spring of 2003, leaving the world's mathematicians to pick up the pieces and decide if he was right. It took mathematicians all of three years to check his work, and the evidence was posted on a preprint server in the form of three book-length papers totalling nearly a thousand pages. 'It's really a great moment in mathematics,' said Bruce Kleiner of Yale, who spent three years helping explicate Perelman's work. 'This is a kind of twentieth-century Pythagorean theorem,' said Barry Mazur of Harvard. 'It changes the landscape.' John Morgan, a mathematician from Columbia, said that the excitement came not from the final proof of the conjecture, which everybody felt was true, but the method, 'finding deep connections between what were unrelated fields of mathematics'. William Thurston of Cornell, himself a Fields Medalist and the author of a deeper conjecture that includes Poincaré's and that is now apparently proved by Perelman, said, 'Math is really about the human mind, about how people can think ef-

The Fields Medal, which is awarded every four years, is supposed not only to reward past achievements but also to stimulate future research; for this reason, it is given only to mathematicians aged forty and younger. Like the Nobel Prize, it grew, in part, out of a desire to elevate science above national animosities. German mathematicians were excluded from the International Congress of Mathematicians in 1924, and, though the ban was lifted before the next congress, the trauma it caused led to the establishment of the Fields Medal in 1936—a prize intended to be 'as purely international and impersonal as possible.'

fectively, and why curiosity is quite a good guide,' explaining that curiosity is tied in some way with intuition.

But at the moment of his putative triumph, Perelman was nowhere in sight. He had simply vanished from the public eye. 'It's very unusual in math that somebody announces a result this big and leaves it hanging,' said Morgan. Also left hanging is the million-dollar award offered by the Clay Institute. Sir John Ball, the fifty-eight-year-old president of the International Mathematical Union, the discipline's influential professional association, went to St Petersburg to meet Perelman and persuade him to accept the prize in a public ceremony at the International Congress of Mathematicians, Madrid, in August 2006. But Perelman told Ball that he had no intention of accepting it. 'I refuse,' he said tersely. The Fields Medal held no interest for him, Perelman explained. 'It was completely irrelevant for me,' he said. 'Everybody understood that if the proof is correct then no other recognition is needed.'

A Vedantic Critique

This, then, is our story of the enigma that Perelman is. What do we make of this strange mathematical genius who refused the highest award—the only mathematician ever to do so? It is only a deeper value of commitment to knowledge for its own sake that can explain the phenomenon. In a world ridden by materialistic values, where compromise and camouflage are euphemistically and dishonestly described as practicality and realism, and genuine idealism is dubbed impracticality, Perelman comes to us as a fresh source of inspiration. Milton speaks of fame as the 'last infirmity of a noble mind'. One cannot help saluting one who can refuse such an invitation to instant renown. This dispassion, or vairagya, is a hallmark of the true aspirant for jnana, knowledge. Only a person who knows human accolade to be ephemeral and all ululation to be the emotional outpouring of confused and intoxicated minds can take such a clear and hard stand. Unless viveka, or discrimination, and vairagya have become established to a great extent, and inner knowledge has dawned, the heart cannot afford to be so bold. In Perelman we see a person who has not formally taken sannyasa, but who remains single and has renounced all attachment to money beyond that needed to meet his very basic necessities. Perelman lives in penury now. He also seems to have no desire to continue in any formal institute, for that would mean being an object of public attention, since he has already achieved so much. Thus the Upanishadic description '*putraiṣaṇāyāśca vittaiṣaṇāyāśca lokaiṣaṇāyāśca vyutthāyātha bhikṣācaryaṁ caranti*; [the ancient sages] renounced their desire for sons, for wealth, and for the world(s), and lived the mendicant's life' seems to have been given with just such people in mind.[2] What is he but a sannyasin—in the truest sense of the term—whose renunciation (*nyāsa*) is comprehensive (*samyak*)? Sri Ramakrishna speaks of *jnanonmad*, 'the madness of knowledge', and gives a graphic description of this state.[3] Grisha Perelman reminds one of this madness. Madness, of course, but what madness! The world would be blessed with a little more of such madness, and a little less of humdrum sanity!

References

1. Much of the biographical information on Perelman presented here is based on articles that appeared in the *New York Times* and the *New Yorker*.
2. *Brihadaranyaka Upanishad*, 4.4.22.
3. M, *The Gospel of Sri Ramakrishna*, trans. Swami Nikhilananda (Chennai: Ramakrishna Math, 2002), 548.

REVIEWS

For review in PRABUDDHA BHARATA,
publishers need to send ***two*** *copies of their latest publications.*

Bhaskaracharya's Bijaganitham

Dr V B Panicker

Bharatiya Vidya Bhavan, Kulapati Munshi Marg, Mumbai 400 007. E-mail: *brbhavan@bom7.vsnl.net.in*. 2006. 198 pp. Rs 120.

Bhaskaracharya's *Bijaganita* records some of the grand achievements of ancient Indian algebraists. It is thus a valuable document for the proper understanding of an important aspect of Indian history. Further, as alluded to in an accompanying article in this issue, an early and imaginative use of the text can strengthen the scientific intellect of our students and sharpen the algebraic skills of budding mathematicians. It is imperative that editions of Bhaskaracharya's *Bijaganita* be available in libraries of schools and colleges and in mathematics libraries, as well as in libraries on Indian history and culture. However, although a few scholars (including S K Abhyankar) have brought out editions of the book in the past, they are not easily available. It is to be hoped that the new Bharatiya Vidya Bhavan publication edited by Dr V B Panicker will enhance the visibility of this great treatise.

In this edition, along with the original Sanskrit verses of *Bijaganita*, Panicker has given the Roman transliteration followed by an English translation of the verses. The mathematical rules and examples have been lucidly explained in standard high-school language, making it smooth reading for students and general readers. Exercises set by Bhaskaracharya have also been completely worked out. However, a few typing errors are jarring: for instance, the symbol for cube-root reads like '3 times square root'. One hopes that a more careful proof-reading will be done for future editions.

Since the book is for the general reader, it is necessary to describe the transliteration scheme. (In any case, the scheme followed in the book differs slightly from the standard one.) There are likely to be many readers uncomfortable with Sanskrit but eager to relate to the original verses. It would be helpful for them if, along with the overall meaning of the stanzas, individual word meanings are also provided; or at least, a glossary of the technical terms is appended (as in S K Abhyankar's edition). After all, even people with a Sanskrit background may not be familiar with all the ancient Indian mathematical terms. It would also be nice if the translation tries to bring out something of the lyrical quality of the original.

For enhancing the pedagogic value of the book, it is desirable that students are shown how the mathematical ideas in *Bijaganita* are linked with important modern concepts. Even if such a discussion is not possible, at least some references could be given which serious students of mathematics could pursue for a deeper appreciation of the treatise. The depth and significance of ancient Indian algebra needs to be brought out not only for students interested in mathematics but also for scholars on Indian history.

In any case, Dr Panicker has made a commendable effort in highlighting an important landmark in Indian mathematical heritage.

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Developments in Indian Philosophy from Eighteenth Century Onwards: Classical and Western

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Centre for Studies in Civilizations, New Delhi. Distributed by Motilal Banarsidass, 41 U A Bungalow Road, Jawahar Nagar, Delhi 110 062. E-mail: *mlbd@vsnl.com*. 2002. xxiv + 416 pp. Rs 1,200.

Padma Vibhushan Prof. K Satchidananada Murty wrote in the foreword to his *Indian Philosophy since 1498*: 'Works on the history of Indian Philosophy tend to give the impression that there had

been either an absence or an eclipse of philosophical thinking in India from the beginning of the sixteenth to the second decade of the nineteenth century. But in fact, this period had been no less creative than the one that preceded it or the one that succeeded it.'

The book under review, which is a commendable survey of Indian philosophical literature from the eighteenth century to the present, confirms Prof. Murty's assertion to be a fact. This is Volume Ten, Part One, of the well-known encyclopaedic *Project of History of Indian Science, Philosophy and Culture*. It is divided into four sections which broadly address twelve fascinating themes. Besides, there is a comprehensive introductory note by Prof. D P Chattopadhyay, general editor of the series, and a scholarly editorial introduction by Prof. Daya Krishna. This high-quality work provides a critical evaluation and objective assessment of the developments in classical Indian philosophical systems—Nyaya, Sankhya, Mimamsa, Vedanta, Dharmashastra, Alankarashastra, and Jaina thought—spanning the duration of the last three centuries.

Section One comprises two chapters, of which the first elucidates the background of developments in traditional Indian thought from the eighteenth century by surveying the relative strengths of various Indian philosophical schools. The second chapter explains how, during the times of crisis in polity and society in the seventeenth and eighteenth centuries, Indian scholars searched for sources of renewal in the sacred texts of Hindus and Muslims by reinterpreting them, thus unravelling their inherent meanings.

Section Two consists of eight chapters. Chapter Three provides some new perspectives on the classical tradition of philosophizing in Indian terms by focusing on the rich diversity of views on the Indian philosophical tradition among Indian writers. The next three chapters elucidate various developments in Mimamsa, Sankhya and Yoga, and Nyaya through a survey of the contributors to these systems of thought. Chapter Seven throws light on the controversy between the Advaitins and non-Advaitins, beginning with Madhusudana Saraswati. Chapters Eight and Nine review the Dharmashastra and Alankarashastra literature, and Chapter Ten deals with some developments in Jaina thought from the time of Yashovijaya.

The sole chapter in Section Three studies the developments in Indian philosophy as written in English after the advent of the British and endeavours to show how Indian writers on philosophy have been influenced by the Western philosophical traditions to which they were exposed by their education. Section Four consists of two chapters, of which the first discusses different problems and issues which have not yet been explored in Indian intellectual history. This chapter further draws our attention to the dire need for a critical evaluation of thinkers of different schools of thought from the eighteenth century onwards and reiterates the need to re-examine all available source material on humanities in Indian languages.

The final chapter—comprising the editor's concluding remarks—is followed by a set of six appendices which contain a rich bibliography of authors in various fields of philosophy, and lists of important Mimamsakas and Naiyayikas, of polemical texts (*vada granthas*) and Dharmashastras, of various Indological series published from the nineteenth century onwards, and of eminent pundits and Indian philosophers of the Western tradition who have contributed to philosophical thought during this period.

Like the other volumes in this project, this volume too is the product of active cooperation of a galaxy of Indian scholars with various specializations such as Prahlada Char, Kutumba Shastri, K T Pandurangi, Thangaswami Sarma, Kishornath Jha, and Achyutanand Das. The editor's work was considerably eased by the bibliographical sources compiled by Karl Potter (*Encyclopedia of Indian Philosophies*) and Thangaswami Sarma (*A Bibliographic Survey of Advaita Vedanta*). Laxman Sastri Joshi's *Dharma Kosha* volumes and A K Srivastava's *Adhunika Sanskrit Kavyashastra* are some of the other important volumes that have aided this project.

Let us hope that the contemporary Indian scholars and academicians interested in Indian philosophy would heed Prof. Daya Krishna's call for a 'sustained long-term programme' to be undertaken by the twenty-first century institutions of learning and research in India which, besides their respective intellectual activities of various dimensions, should also forge 'a living link between that which was achieved in the past and that which is being done in the present so that it all may become an active ingredient in the building of the future'.

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REPORTS

Sri Gopal Krishna Gandhi lights the lamp inaugurating Seva Pratishthan's platinum jubilee (above); the new building (right)

Seva Pratishthan Celebrates 75 Years

A young Hindu monk thinking of implementing maternity and child-care projects in the bastions of orthodox Calcutta was unthinkable. Another unthinkable: that Indian women, accustomed to delivering babies at home and in private, would agree to participate. And a third: convincing the senior monks at Belur Math of his plans. But the unthinkable became reality. Swami Dayananda (1892–1980), a disciple of Sri Sarada Devi, conceived of an institution for serving newborn children and their mothers when he was serving as a preacher of Vedanta in San Francisco. The health and happiness of American babies and children and the health-care facilities available to them highly impressed him; at the same time, he was saddened to think of the unhealthy condition of babies and their mothers in India. The Swami wanted to try to ameliorate their suffering.

On 24 July 1932, in rented premises in Bhowanipur, Kolkata, the Shishu Mangal Pratishthan was founded, with financial assistance from Mrs Helen Rubel, an American student of Vedanta, and volunteer support of three professionally trained nurses—an Indian, an American, and a German. The new institution began working to provide instruction in hygiene, and efficient pre- and post-natal care to mothers and their babies. Much of the initial service was provided through home visits; the institution started with only seven beds.

The institution shifted to its present location in 1939, where a hospital with 50 beds was constructed. In 1956, the institution became a general hospital, with a school of nursing, and in 1957, was renamed **Ramakrishna Mission Seva Pratishthan**.

The hospital developed a wing of research in medicine and post-graduate teaching, known as the Vivekananda Institute of Medical Sciences, which was accredited by Calcutta University in 1963. In 2005, the Ma Sarada College of Nursing was added.

On 24 July 2007, Srimat Swami Gahananandaji Maharaj, President, Ramakrishna Math and Ramakrishna Mission, inaugurated a new seven-storey hospital extension, called the Adhish Chandra Sinha Memorial, in the presence of a large number of monks, novices, doctors, nurses, non-medical staff, donors, and dignitaries. The new building will accommodate the following: reception and modern general emergency department; paediatric wards with advanced treatment facilities including an intensive neonatal care unit and facilities for treatment of surgical, eye, ENT, and orthopaedic problems of children; diagnostic facilities for outdoor patients; a canteen for patients, staff, and visitors; a waiting hall for patient parties; a school and college of nursing; Swami Dayananda Auditorium; and an additional 60 beds for children, bringing the total number of children's beds up to 130.

The inaugural function of the Platinum Jubilee, held after the new building was opened, was inaugurated by Sri Gopal Krishna Gandhi, Governor of West Bengal. About 600 monastics and 2,500 devotees attended the function.

Relief

Heavy rains in the month of July caused severe flooding in parts of Gujarat, Orissa, and West Bengal. Centres of the Ramakrishna Math and Rama-

Flood relief, Gujarat: from left, filling relief packs, relief materials ready for distribution, awaiting supplies

krishna Mission in these states began relief operations immediately. Relief details follow.

Limbdi: 2,980 kg bajra, 2,225 kg rice, 1,377 kg dal, 148 kg tea, 1,482 kg sugar, and 306 kg edible oil to 306 flood-affected families of 4 villages in Surendranagar district; **Rajkot**: 1,113 kg khichri, 1,500 theplas (a kind of roti), 1,855 kg wheat flour, 489 kg vegetables, 185 kg edible oil, 98 kg spices, 416 kg salt, 216 kg snacks, 62 kg tea powder, 371 kg sugar, 326 chadars, 2,226 candles, 371 matchboxes, and 371 plastic sheets to 671 families of low-lying slum areas in Rajkot city and one village each in Jamnagar and Rajkot districts; **Bhubaneswar**: 5,325 kg chira, 530 kg sugar, 1,062 kg salt, 1,080 biscuit packets, 2,120 candles, and 2,120 matchboxes to 5,923 persons belonging to 7 villages of Panasa gram panchayat in Jajpur district; **Belgharia**: 11,794 plates of cooked khichri, 20,170 kg chira, 2,021 kg sugar, 18,715 biscuit packets, 407,095 halazone tablets, 150 kg bleaching powder, and 60 kg lime to flood-affected persons belonging to 22 villages of Sabang, Patashpur, and Narayangarh blocks in East and West Medinipur districts; **Contai**: 8,630 kg chira, 1,600 kg sugar, 1,920 biscuit packets, and 400 kg bleaching powder to 17,260 persons of 18 villages of Patashpur block in East Medinipur district; **Ichapur**: 3,709 plates of cooked food, 3,200 kg rice, 71,600 kg chira, 5,700 kg sugar, 179 kg gur, 23,040 biscuit packets, 377 kg milk powder, 280,000 halazone tablets, 5,000 vials of Zeoline, and 4,125 kg bleaching powder to 79,477 persons, and medical treatment to 2,182 persons belonging to 55 villages of Khanakul-1, Khanakul-2, Arambag, and Ghatal blocks in Hooghly and West Medinipur districts; **Kamarpukur**: 12,500 kg chira, 3,140 kg gur, 5,280 biscuit packets, 110,000 halazone tablets, 1,250 kg bleaching powder, and 9,800 kg lime to 63,918 persons belonging to 20 villages of Khanakul-2 block in Hooghly district; **Medinipur**: 156,900 kg chira, 39,200 kg gur, 240,000 halazone tablets, and 250 kg bleaching powder to 159,897 persons of Ghatal municipality, Ghatal sub-division, and Narayangarh and Sankrail blocks in West Medinipur district; **Ramharipur**: Cooked food to 125 inhabitants of Roniara village, Bankura district, for three days; **Saradapitha**: 5,300 kg rice, 1,200 kg dal, and 225 kg biscuits to 6,529 persons belonging to 14 villages of Udaynarayanpur block in Howrah district; **Tamluk**: 95,900 plates of cooked food, 11,000 kg chira, 2,200 kg sugar, and 120,000 halazone tablets to 11,000 persons of Pingla and Sabang blocks in West Medinipur district.

The following centres of the Ramakrishna Math and Ramakrishna Mission distributed various items to people in need, as indicated. **Aalo**: 1,147 blankets and 1,121 mosquito-nets; **Belgaum**: 107 wheelchairs, 227 tricycles, 94 crutches, 37 walking sticks, 99 hearing aids, 159 slates for the blind, 155 blind canes, 3 cervical collars, and 3 rollators (rolling walkers); **Nagpur**: 410 school uniforms and stationery; **Puri Math**: 108 saris and 100 school uniforms; **Rahara**: 50 kg baby food, 195 utensil sets, 101 shirts and pants, 95 frocks, 19 dhotis, 187 saris, 55 lungis, 55 vests, 91 mosquito nets, and 415 tiffin packets.

Ramakrishna Mission Boys' Home, **Rahara**, built 3 toilets and sank a tube well at Raipara Parashpur Char Colony in Murshidabad district.

Ramakrishna Mission, **Batticaloa**, continued relief operations among thousands of families who have moved to Batticaloa district as a result of ethnic disturbances in Sri Lanka.

The ashrama provided 9,663 kg vegetables, 1,149 kg gram, 472 kg green gram, 1,076 kg cowpea, 312 kg soya, 4,034 coconuts, 175 kg garlic, 267 kg tamarind, 82 litres coconut oil, 1,918 kg salt, 78 kg cumin seeds, 42 kg mustard, 432 kg chili powder, 433 kg tea powder, 516 kg sugar, 750 litres kerosene oil, 150 T-shirts, 200 children's clothes, and 150 saris to 2,390 persons. 53 persons received medical treatment.

The Batticaloa centre has completed construction of all 116 houses taken up for construction for families whose houses were destroyed in the 2004 tsunami; the last 30 houses were handed over to the beneficiaries in August.